

# Microscopically motivated modeling of ferroelectric materials

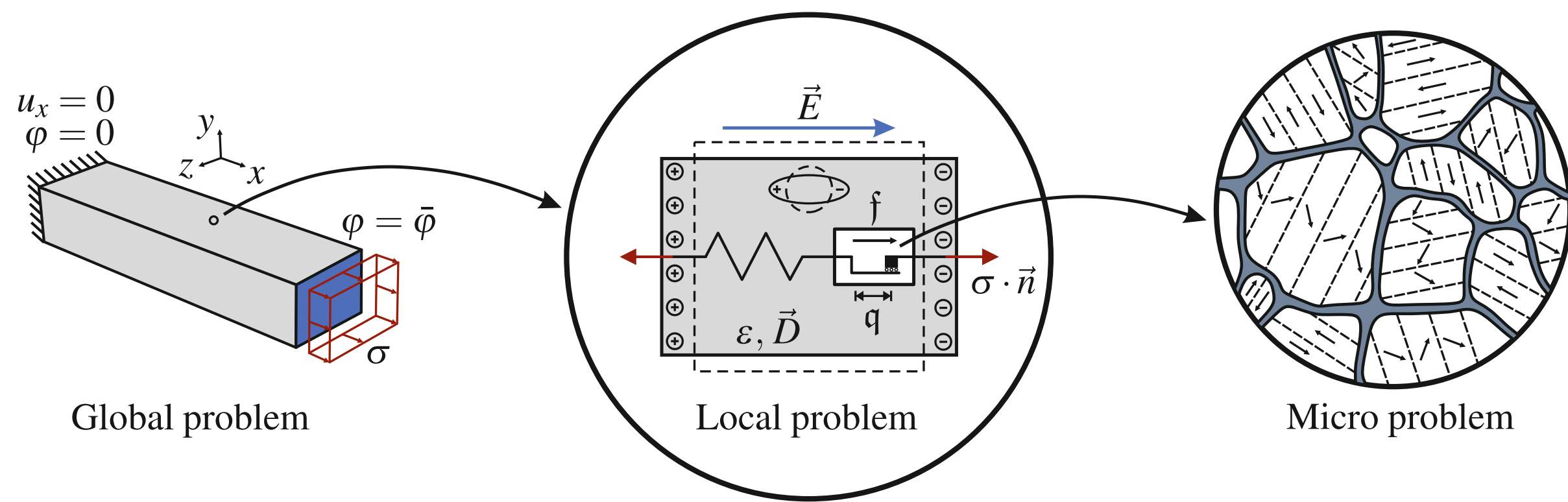
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## Keynotes:

- Variational formulation of a thermodynamically consistent continuum theory for electro-mechanically coupled problems
- Representation of domain switching effects by microscopically motivated internal state variables inside a phenomenological material theory
- Capability to reproduce experimentally measured macroscopic material behavior of PZT ceramics

## Variational modeling framework

### Modeling levels under consideration



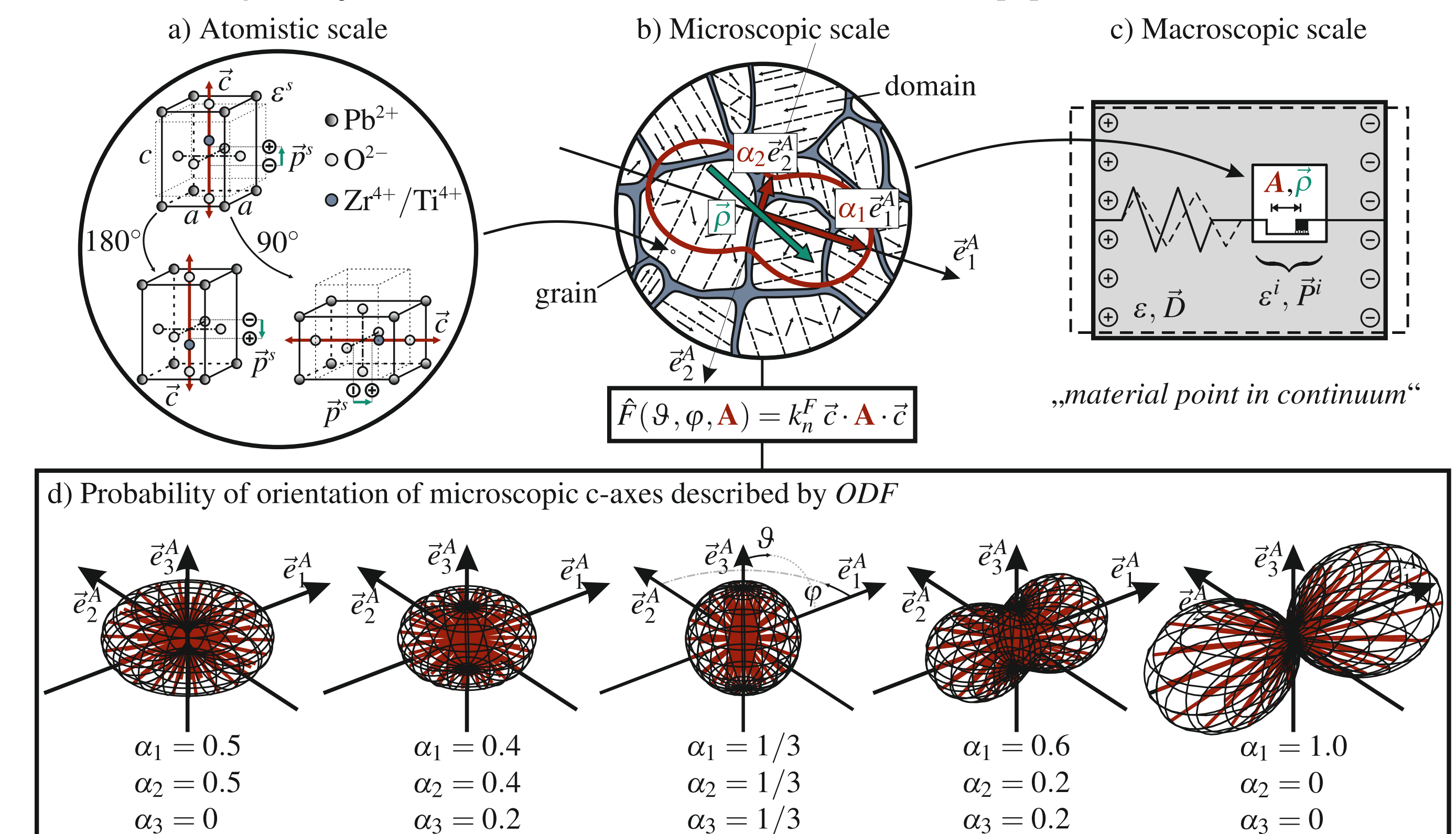
- Electro-mechanically coupled boundary value problem
- Solved by five-field finite element formulation
- Local material point element
- Phenomenological approach for representation of reversible and dissipative material behavior
- Polycrystalline microstructure
- Grains subdivided in domains with intrinsic polarization state
- Switchability of domains caused by external electric fields or mechanical stresses

### Variational formulation of the global boundary value problem and FEM

$$\hat{\Pi}^{HW}(\varepsilon, \bar{D}, \sigma, \bar{u}, \varphi) = \int_B W(\varepsilon, \bar{D}) dV + \int_B \nabla \varphi \cdot \bar{D} dV + \int_B \sigma : (\nabla^s \bar{u} - \varepsilon) dV + \Pi^{ext}(\bar{u}, \varphi)$$

$\square$   $[\varepsilon_{11}, \varepsilon_{22}, \gamma_{12}, D_1, D_2, \bar{\sigma}_{11}, \bar{\sigma}_{22}, \bar{\sigma}_{12}, u_1, u_2, \varphi]$   
 $\bullet$   $[u_1, u_2, \varphi]$   
 $\square$   $[\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \gamma_{12}, \gamma_{23}, \gamma_{13}, D_1, D_2, D_3, \bar{\sigma}_{11}, \bar{\sigma}_{22}, \bar{\sigma}_{33}, \bar{\sigma}_{12}, \bar{\sigma}_{23}, \bar{\sigma}_{13}, u_1, u_2, u_3, \varphi]$   
 $\bullet$   $[u_1, u_2, u_3, \varphi]$

### Microscopically motivated internal state variables [1]



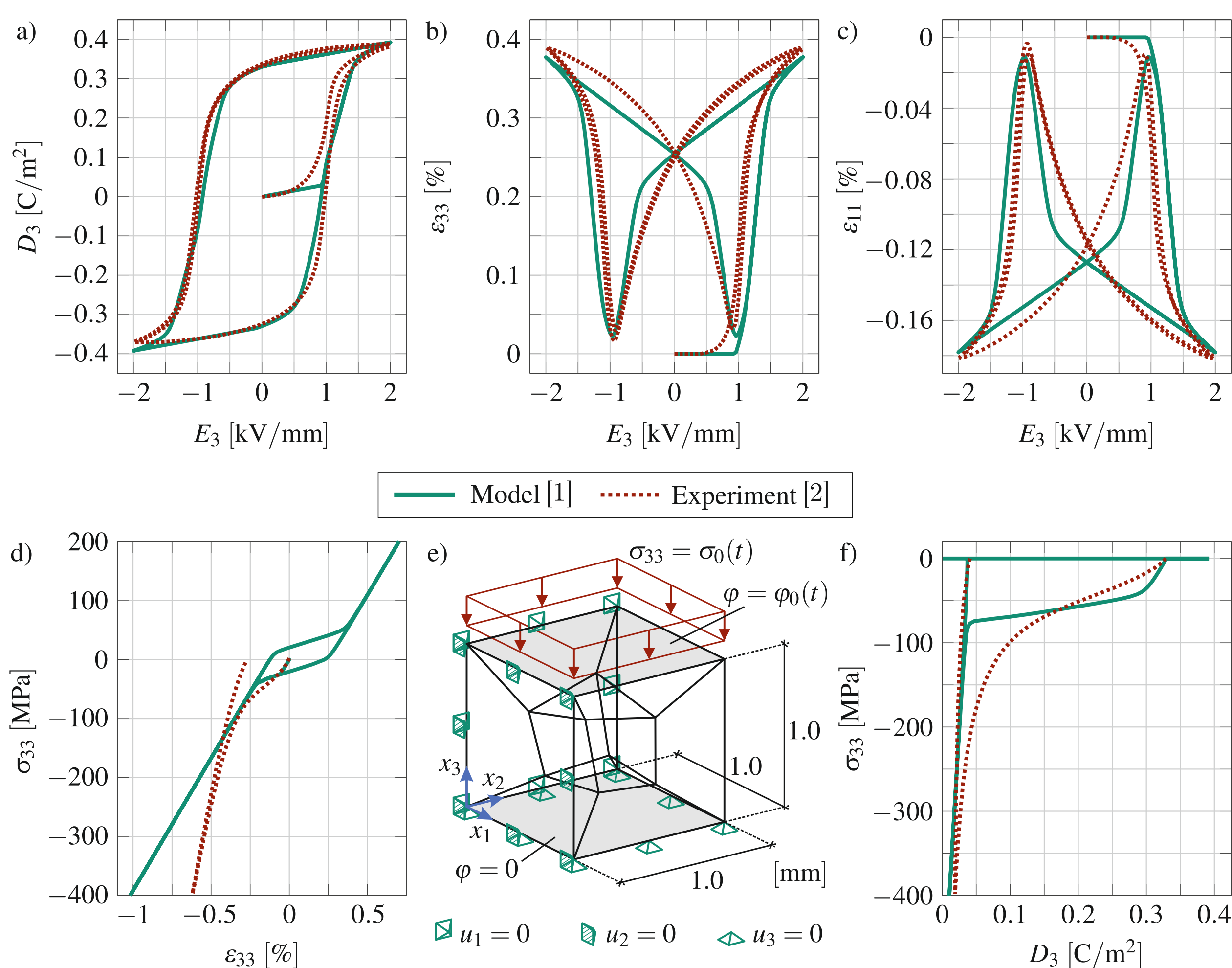
### Incremental variational update of the internal state variables

$$q_n \xrightarrow{t_n} q \xrightarrow{t} q = \text{Arg} \left\{ \inf_q \left\{ \psi(\varepsilon, \bar{D}, q) - \psi_n(\varepsilon_n, \bar{D}_n, q_n) + \sup_{f^q, \gamma} \left\{ f^q \cdot (q - q_n) - \gamma (f^q - f_c) \right\} \right\} \right\}$$

$\Delta t \phi((q - q_n) / \Delta t)$

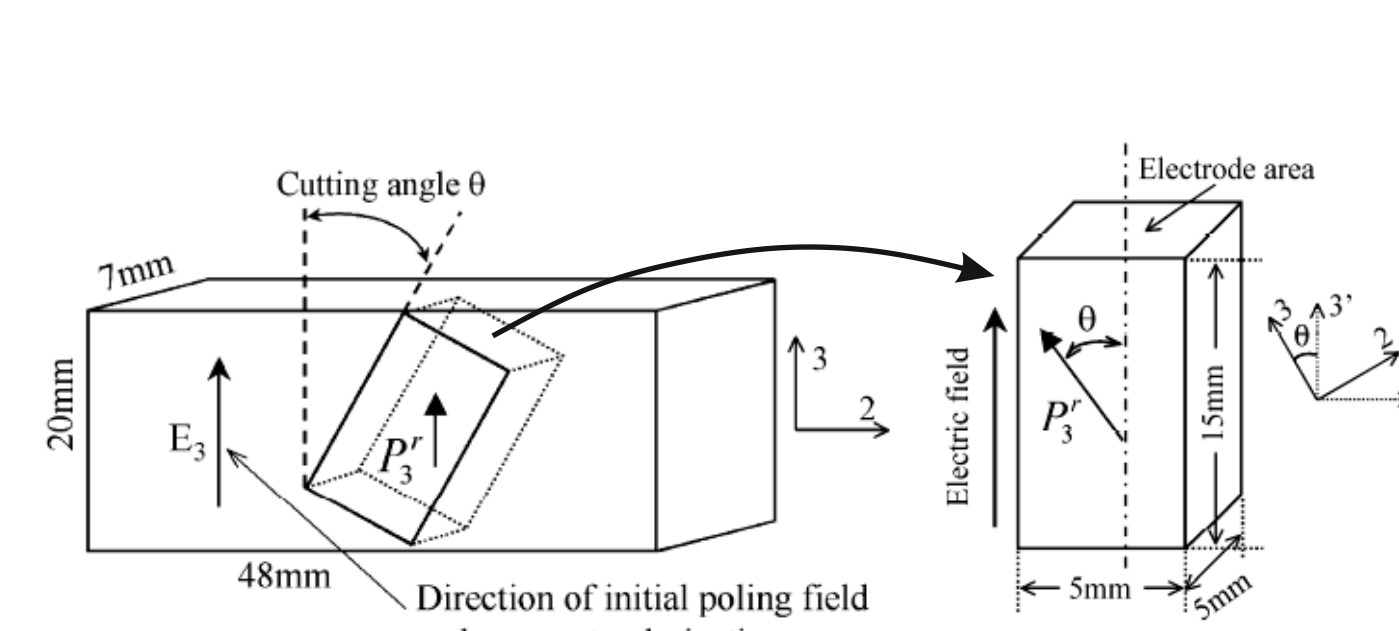
## Numerical results and comparison to experiments

### Characteristic hystereses of PZT

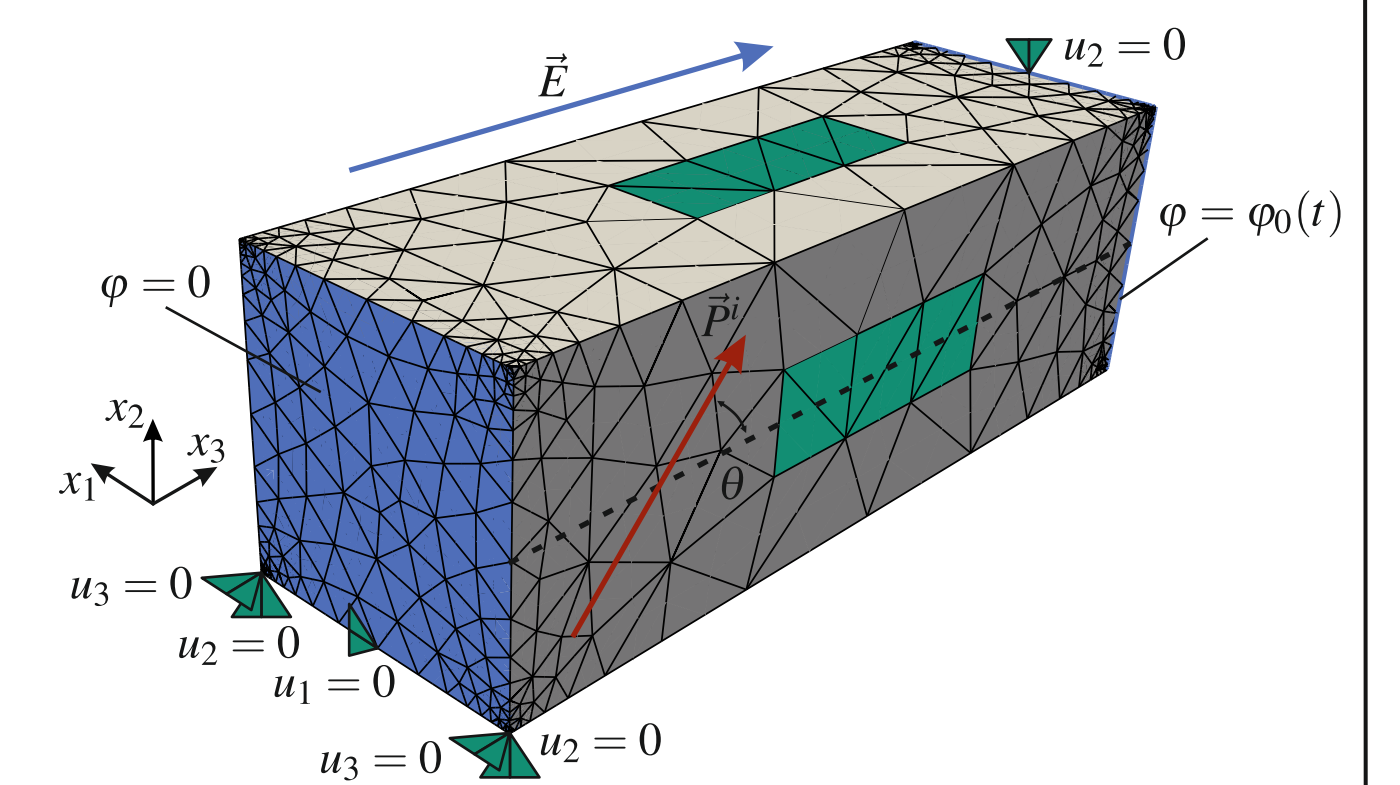


### Polarization rotation test

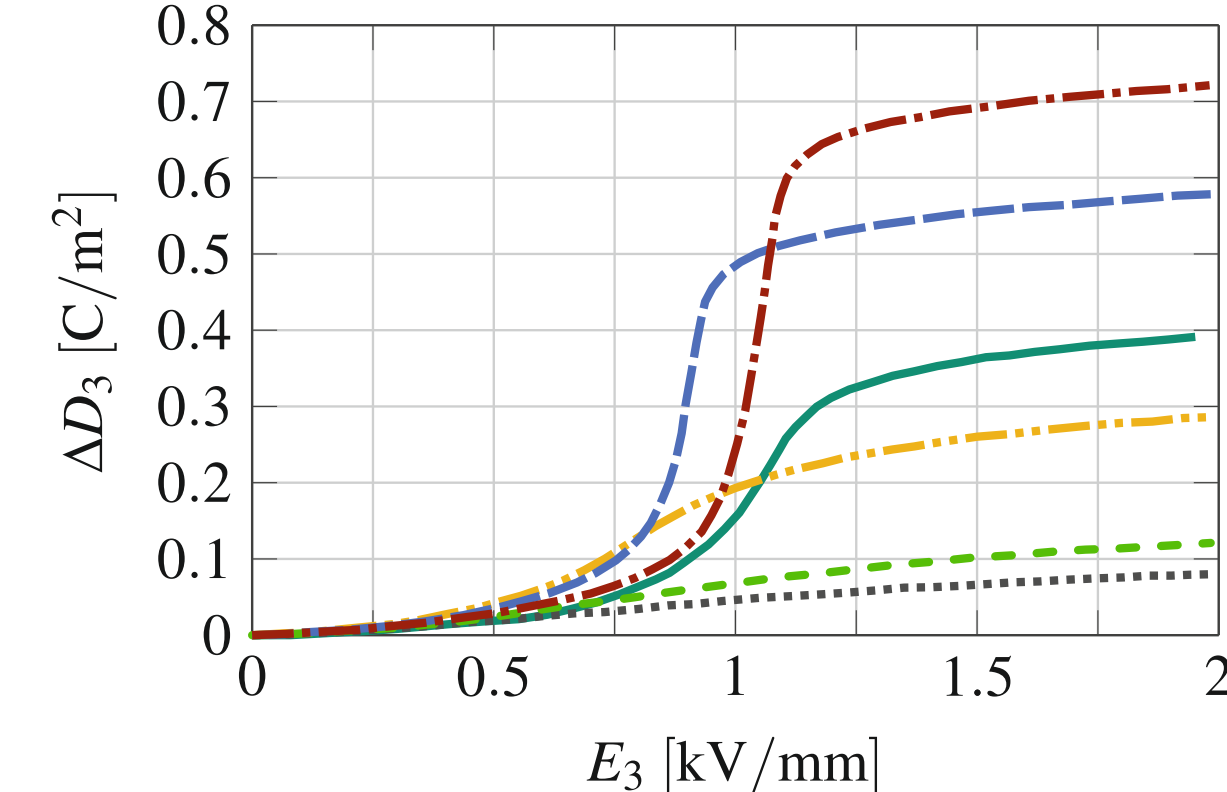
#### Specimen preparation before the experiment [3]:



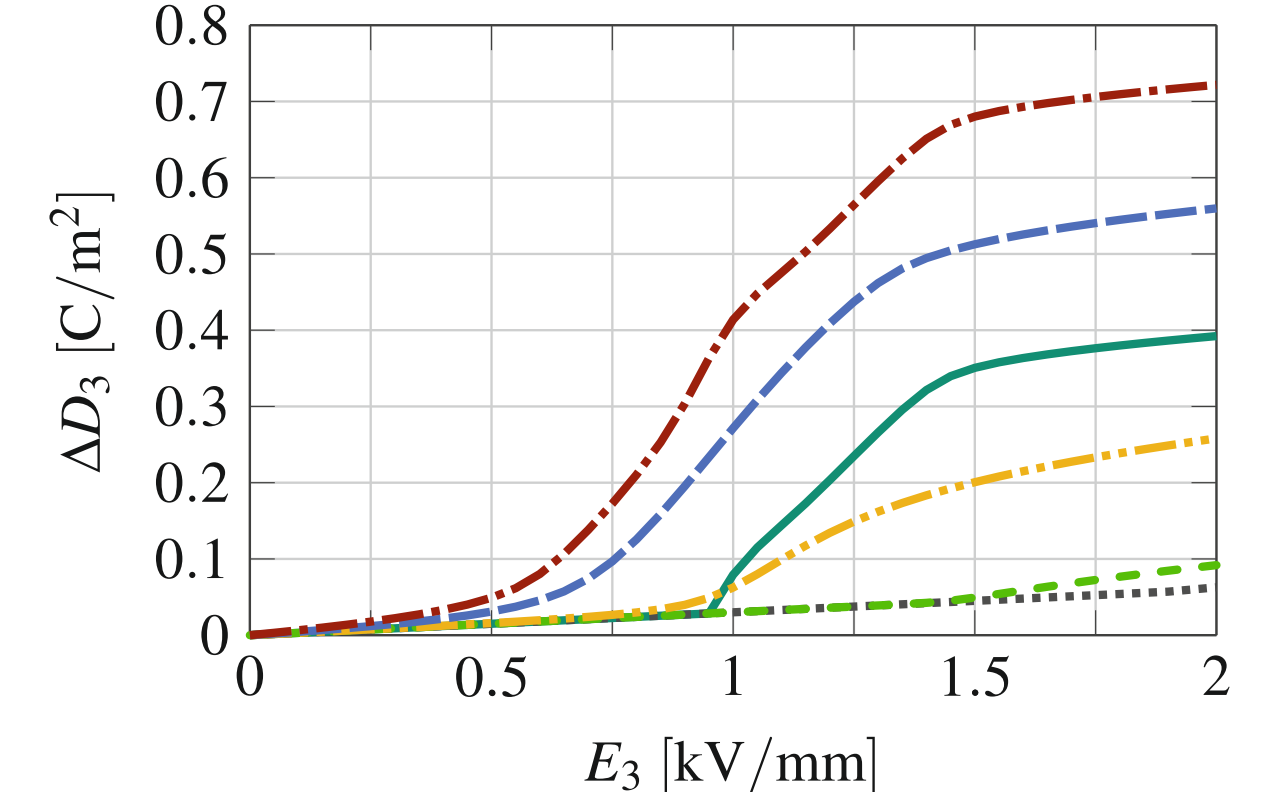
#### Finite element model:



#### Experimental results [3]



#### Simulation results



## Summary

- Numerically stable methods were developed and implemented in an in-house finite element code in MATLAB
- The model is able to reproduce the characteristic material behavior of PZT ceramics, even under complex loading scenarios
- Realistic simulations of the initial poling processes of piezoceramics can make a decisive contribution to the development of fail-safe actuator systems



## Next steps

- Incorporation of the effect of flexoelectricity into the variational modeling framework
- Numerical investigations to the influence of the flexoelectric effect on the poling process in ferroelectric materials
- Gaining insights into the design rules of electrode layouts in order to take advantage of the flexoelectric effect technically

[1] V. Mehling, C. Tsakmakis, D. Gross: Phenomenological model for the macroscopic material behavior of ferroelectric ceramics, J. Mech. Phys. Solids 55 (10) (2007) 2106-2141.

[2] D. Zhou: Experimental investigation of non-linear constitutive behavior of PZT piezoceramics, PhD Thesis, FZ Karlsruhe, 2003

[3] D. Zhou, M. Kamlah, B. Laskewitz: Multi-axial non-proportional polarization rotation tests of soft PZT piezoceramics under electric field loading, Proc. SPIE 6170, Smart Structures and Materials, 2006