Karlsruhe Institute of Technology

Institute for Applied Materials

Plasticity

Lecture for “Mechanical Engineering” and “Materials Science and Engineering”
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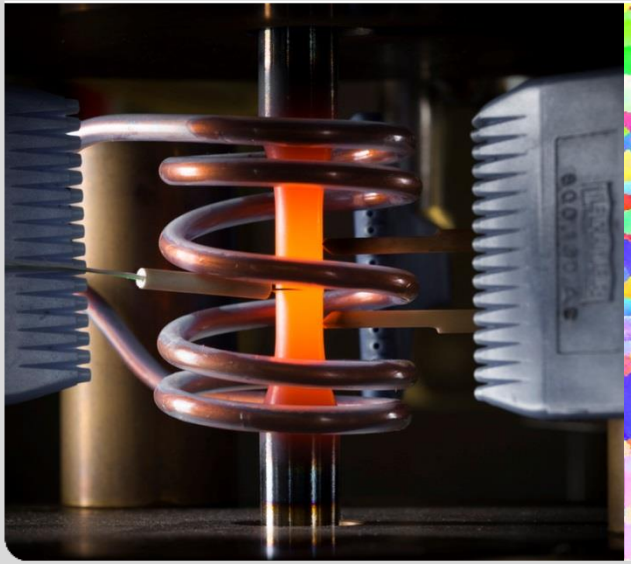
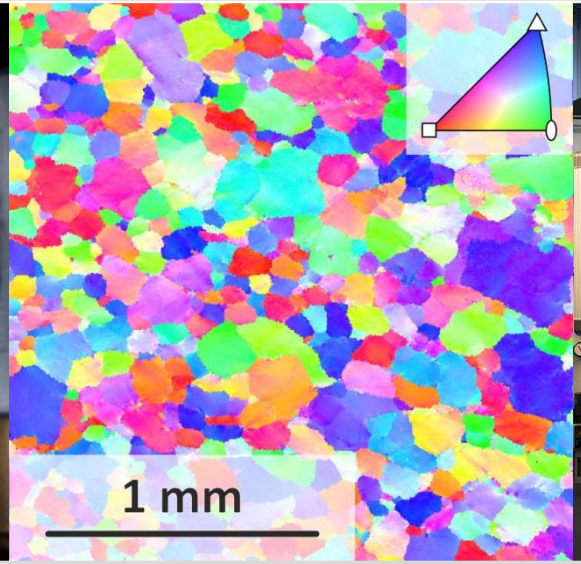




Topics

- Macroscopic plastic deformation: stress-strain-curve in tensile tests
  - Engineering strain, engineering stress
  - Elastic deformation: stiffness/compliance
  - Elastic-plastic deformation: offset strength
  - Localized plastic deformation I: ultimate tensile strength, uniform strain
  - True strain, true stress, work/strain-hardening
  - Considère criterion
  - Localized deformation II: pronounced yielding and Lüders bands

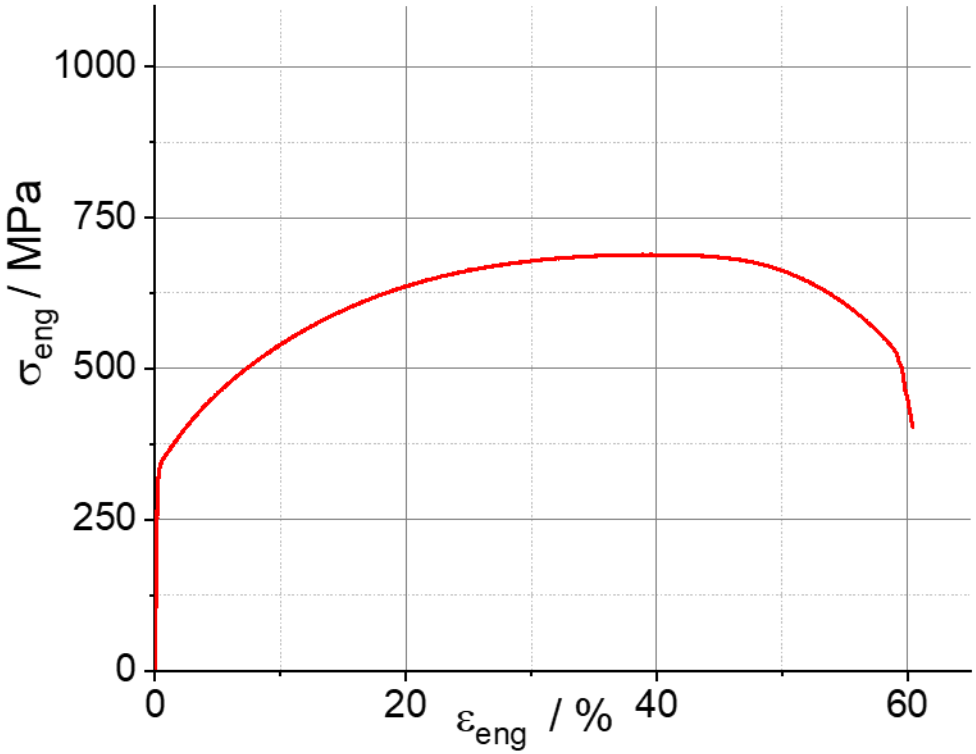
Revision from Other Lectures

- How is a tensile test done?
  - Which sample shapes are usually used in tensile tests?
  - Which quantities are typically controlled during the tests?
  - Which quantities are usually detected?

Plasticity in Metals, Alloys and Intermetallics

- **Metallic materials are very ductile even tough being crystalline!**
- The chapter will deal with the macroscopic interpretation of simple mechanical tests.

example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature



A. S. Tirunilai: „Peculiarities of deformation of CoCrFeMnNi at cryogenic temperatures“, Journal of Materials Research 33 (2018) 3287-3300

Stages of Tensile Tests

- Stages of the deformation during the tensile test:
  - **Linear-elastic deformation** (reversible (almost) without internal friction)
  - **Elastic-plastic deformation** (partially reversible, partially irreversible)
  - **Elastic-plastic deformation with necking** (localization of plastic deformation)
- Mostly presented using engineering quantities:
  - **Engin. strain** $\varepsilon_{\mathrm{eng}} = \frac{\Delta l}{l_0}$ (sometimes "total strain")
  - **Engin. stress** $\sigma_{\mathrm{eng}} = \frac{F}{A_0}$ (sometimes "nominal stress")

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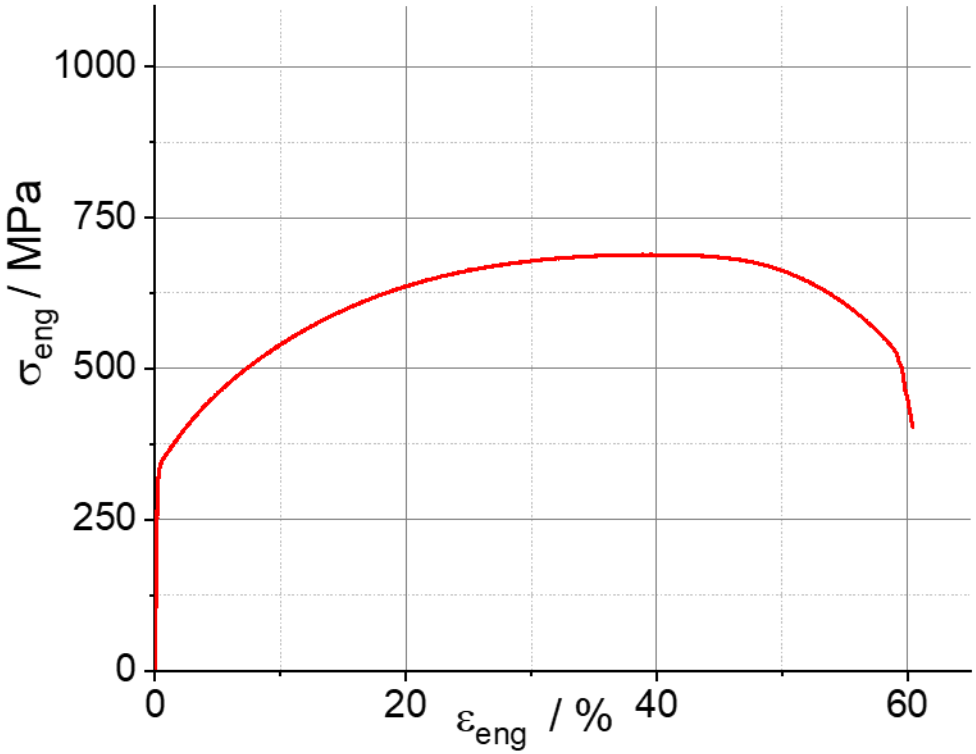

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Linear-Elastic Deformation

- The linear-elastic region can be described by Hooke's law: $\sigma_{\text{eng}} = E \cdot \varepsilon_{\text{eng}}$.
- E is the **resistance against elastic deformation** and called **Young's modulus**.
- In case of polycrystals, E corresponds to an averaging of the compliance/stiffness constancies weight by the orientations distribution function (see. Ch. 3).
- E is one particular stiffness property. Depending on the loading conditions, there are also other stiffness properties, like for example G with $\tau = G \cdot \gamma$ or K with $p = -K \cdot \frac{\Delta V}{V}$.

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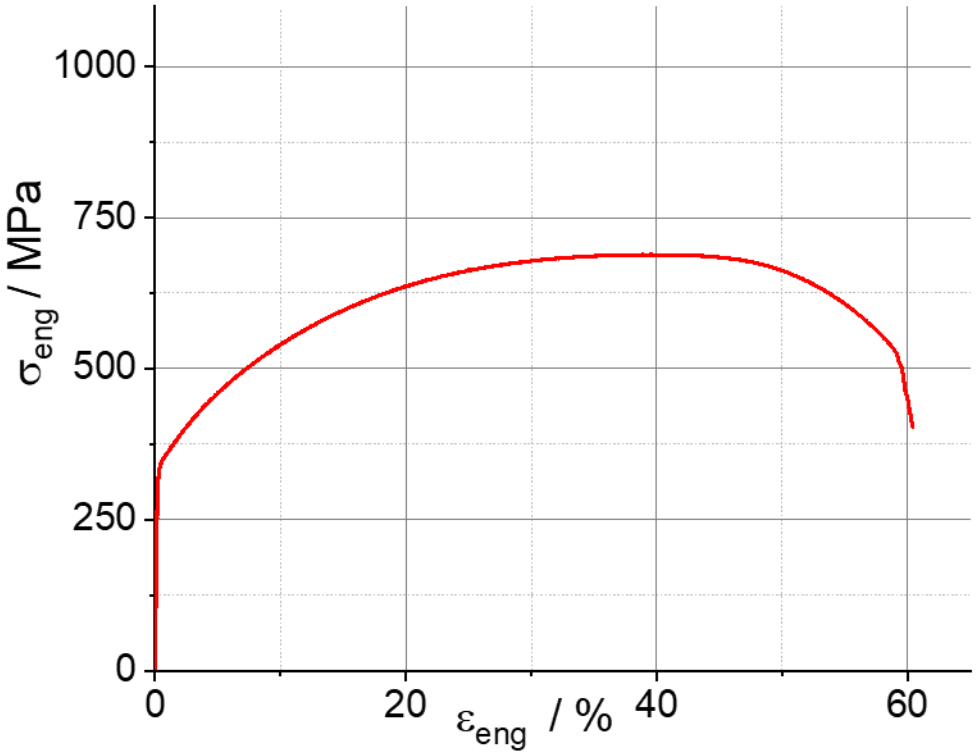

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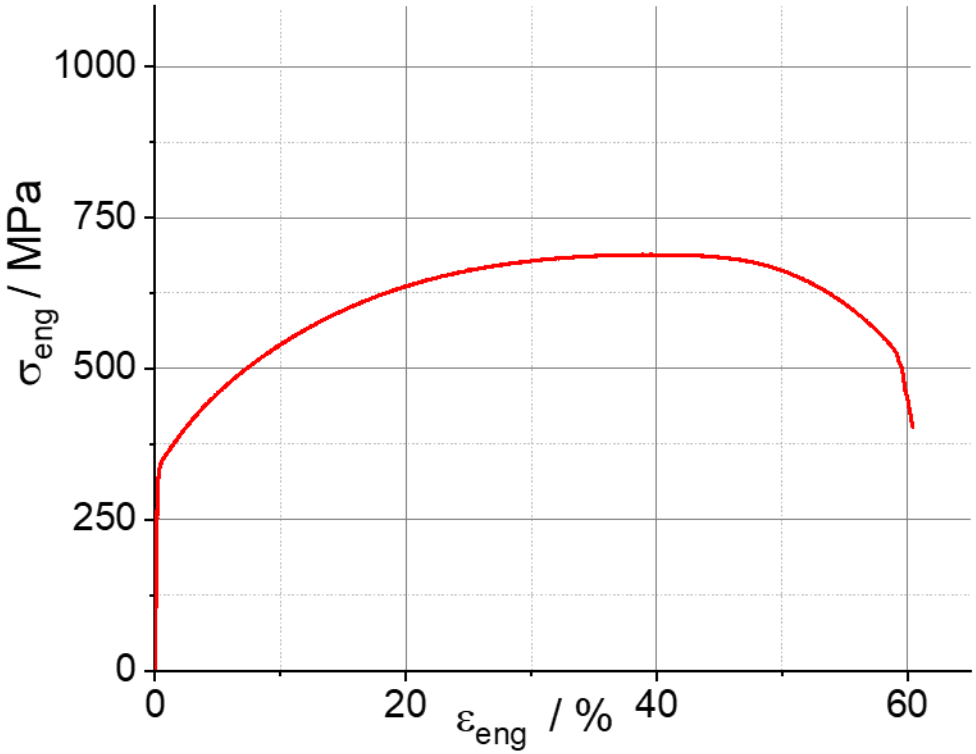
Elastic-Plastic Deformation

- In the elastic-plastic region, some portion of the deformation is irreversible.
- The onset of plastic deformation is called **yield strength** σ_y and determines the **resistance against plastic deformation**.
- In the case of rather continuous stress-strain curves **without pronounced yielding**, **offset strength** is used to describe onset of plastic deformation. For example $\sigma_{0.2}$ is the **resistance against** 0.2% **plastic strain**.
- Anyhow, **both quantities indicate macroscopic strength**. **Microscopic plasticity is taking place also at stresses below** σ_y **and of course** $\sigma_{0.2}$.

example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature



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Elastic-Plastic Deformation

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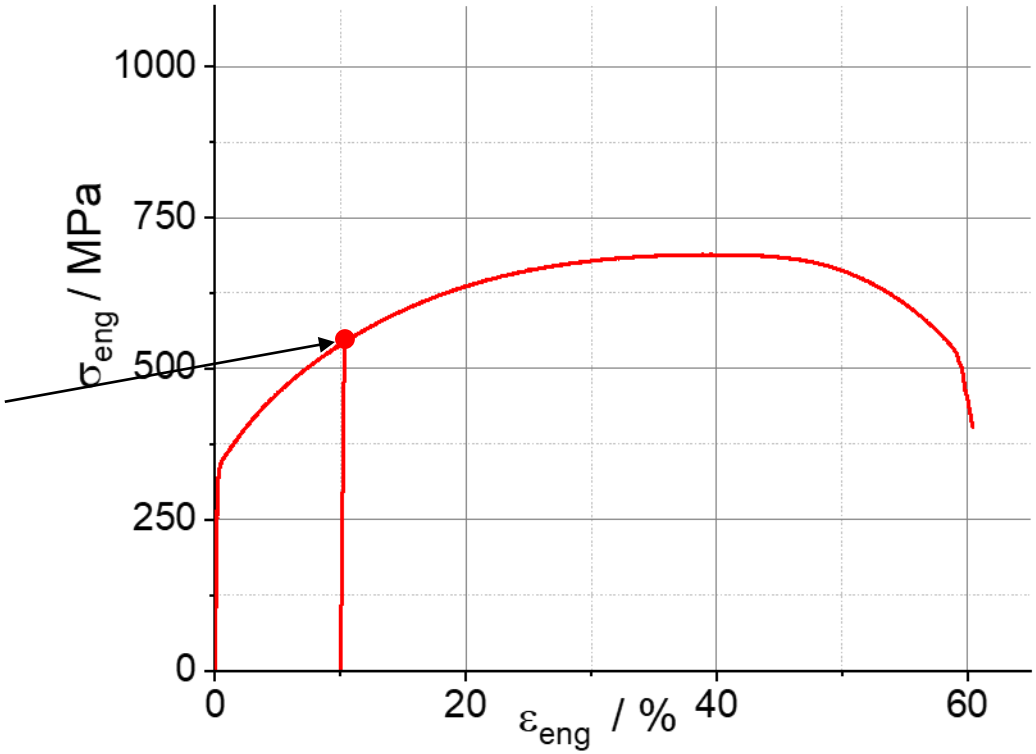

- During unloading, **the elastic deformation *almost** resets**.
- **Onset of plasticity during re-loading is observed at a higher stress in comparison to the initially undeformed material.** This is called strain- or **work-hardening**.

* Bauschinger effect

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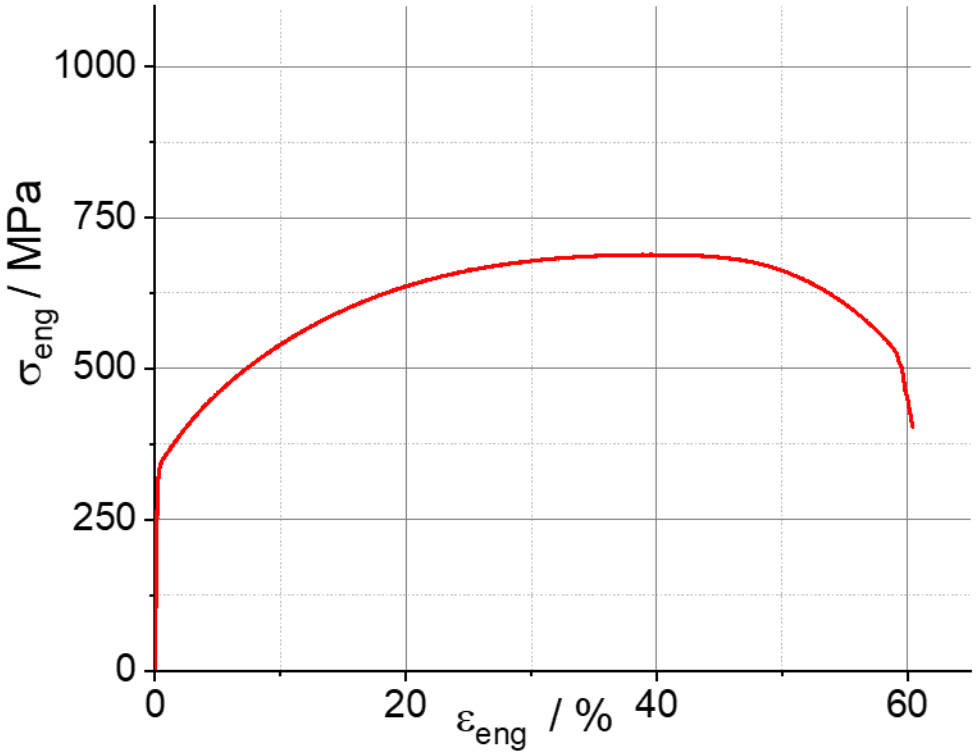
Necking

- **Ultimate tensile strength σ_m** determines the **maximum engineering stress** that the sample can resist. Beyond σ_m, the sample exhibits **necking** and the **plastic deformation becomes localized** to a small portion of the gauge length.
- Plastic strain which achieved up to σ_m is called **uniform elongation (common but imprecise; better to use uniform strain)**.
- Necking occurs due to the **mechanical-geometrical instability** of the tensile test.

example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature



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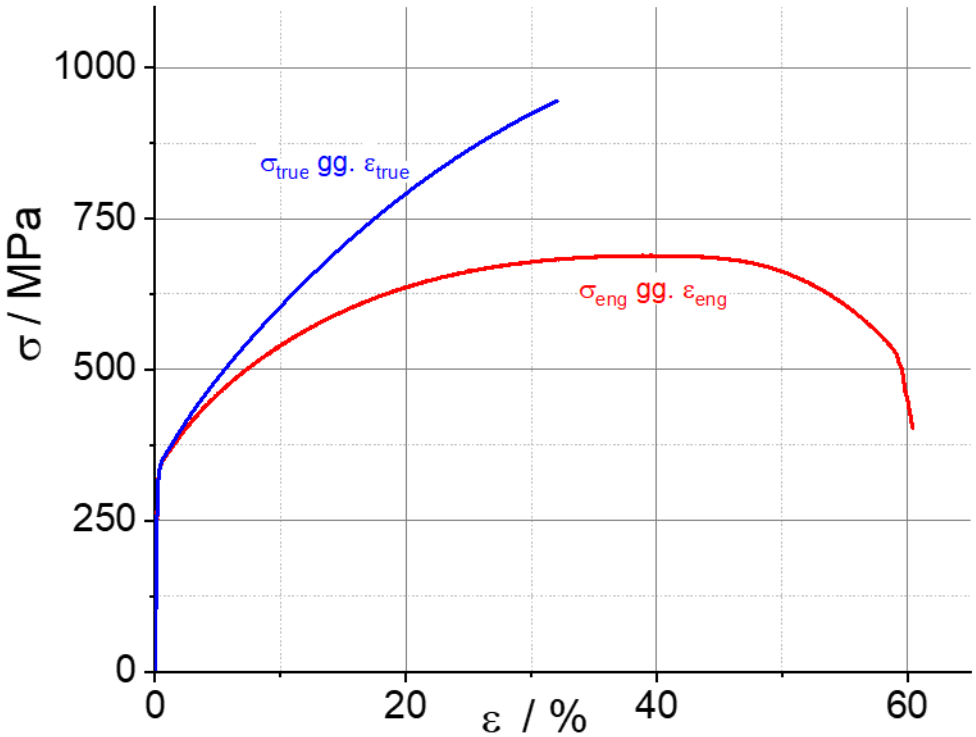
True Stress, True Strain

- For further analysis, the introduction of instantaneous stresses and strains is useful, true strain and true stress:
  - $\mathrm{d}\varepsilon_{\mathrm{true}} = \frac{dl}{l}$, hence, $\varepsilon_{\mathrm{true}} = \ln\frac{l}{l_0}$
  - $\sigma_{\mathrm{true}} = \frac{F}{A}$
- Assuming uniform elongation (prismatic sample shape) and volume conservation:
  - $V = A_0 \cdot l_0 = A \cdot l = \mathrm{const.}$
  - hence, $A = \frac{A_0}{1+\varepsilon_{\mathrm{eng}}}$

gives

- $\sigma_{\mathrm{true}} = \sigma_{\mathrm{eng}}(1+\varepsilon_{\mathrm{eng}})$
- $\varepsilon_{\mathrm{true}} = \ln(1+\varepsilon_{\mathrm{eng}})$

example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature



Note that true strains can be added while engineering strains cannot.

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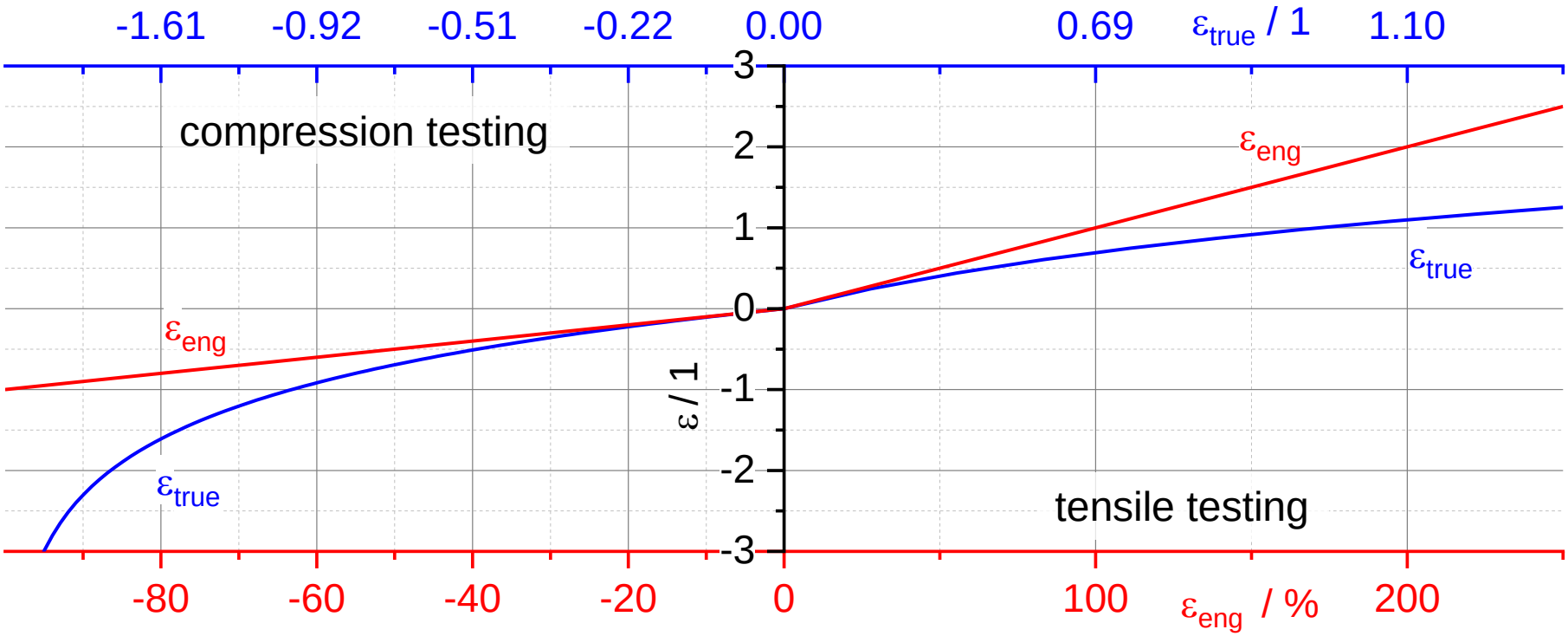
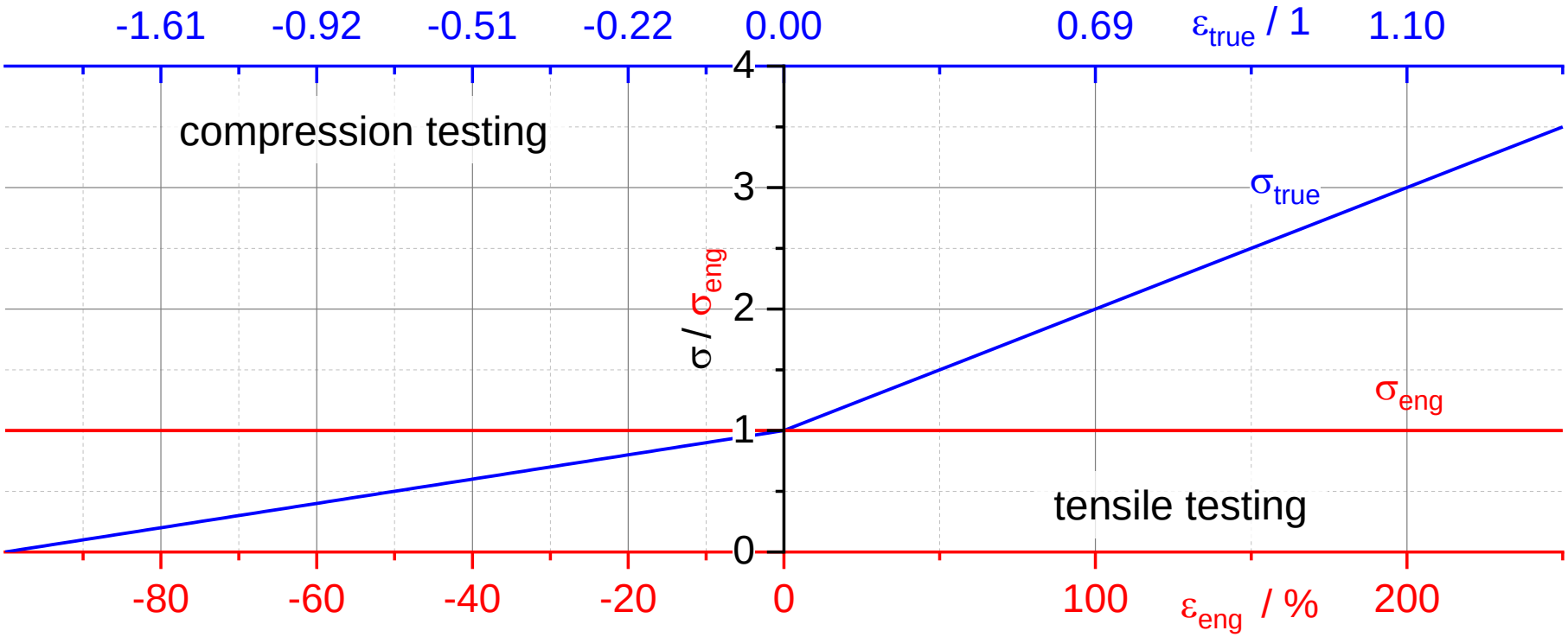
True Stress, True Strain

Direct comparison of ε_{true} and ε_{eng} as a function of ε_{eng}.



True Stress, True Strain

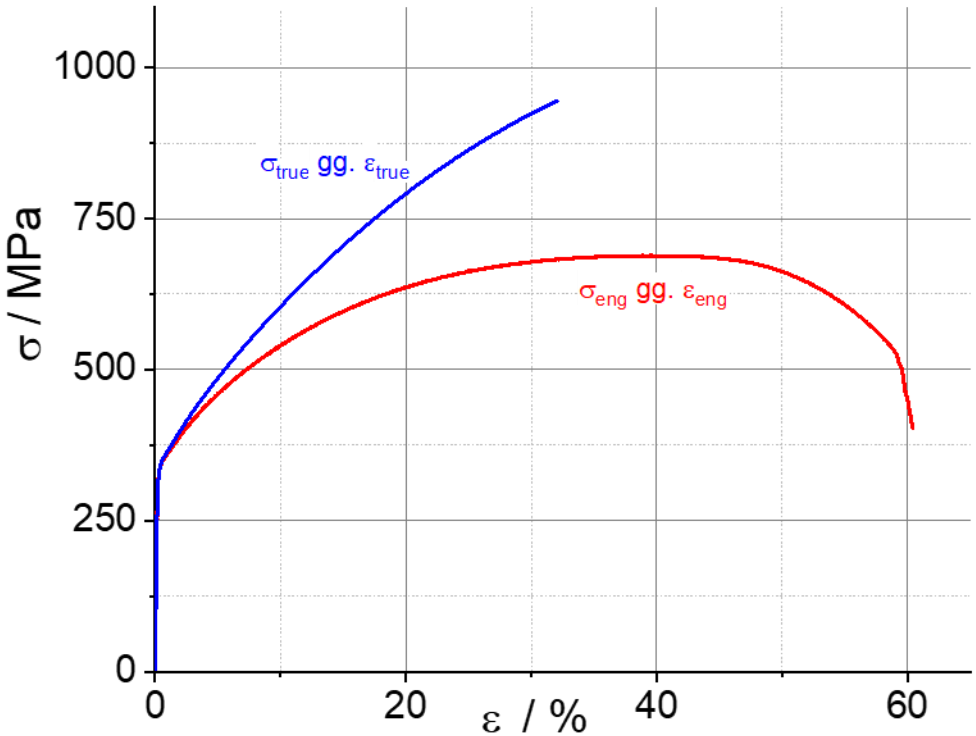
Direct comparison of σ_{true} and σ_{eng} as a function of $\varepsilon_{\mathrm{eng}}$.



True Stress, True Strain

- **Volume conservation is an assumption not a prerequisite**, also for plastic deformation!
- Strictly, **volume conservation** requests a Poisson's ratio of $\nu = 0.5$. For the elastic region, ν is typically about 0.3, after yielding it continuously increases to 0.5. **The deviations are usually neglected when converting from engineering to true quantities**.
- In the necking region, the analytical conversion of engineering to true quantities is not possible anymore. You need to locally resolve strain and stress, for example by DIC.

example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature



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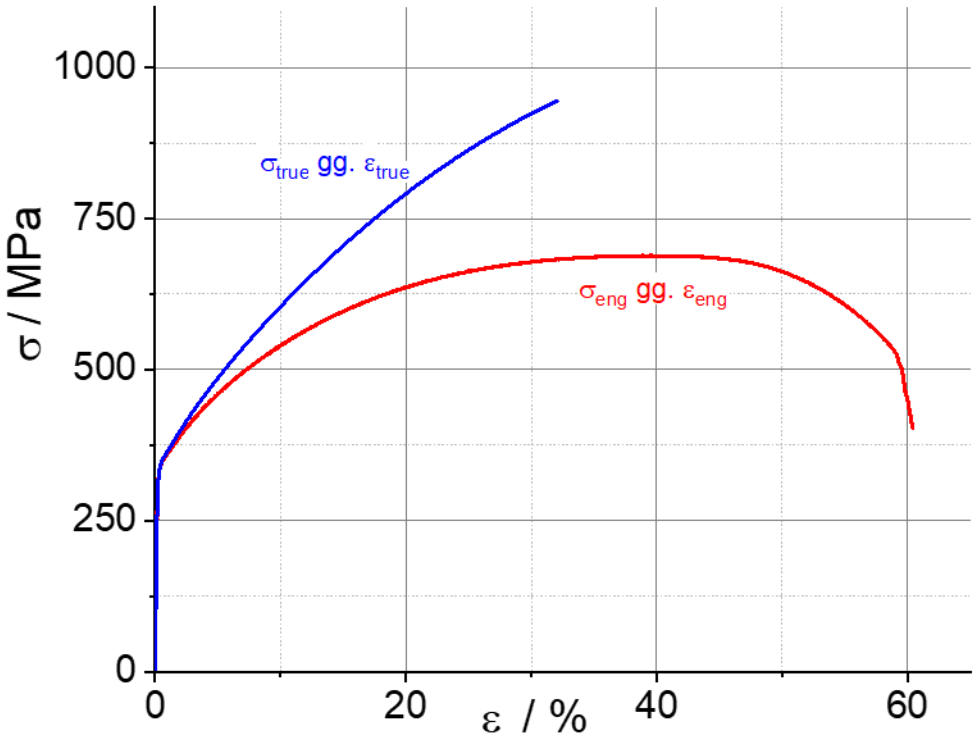
Stability vs. Instability

example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature

- Loading of a sample in tensile geometry leads to two different contributions to the force needed:
- **Geometrical softening**: during loading, the cross section of the sample becomes smaller. In case of nil resistance against further deformation, the force needed to deform would become smaller.
- **Intrinsic strengthening**: The material itself work-hardens; its yield stress is continuously increasing.



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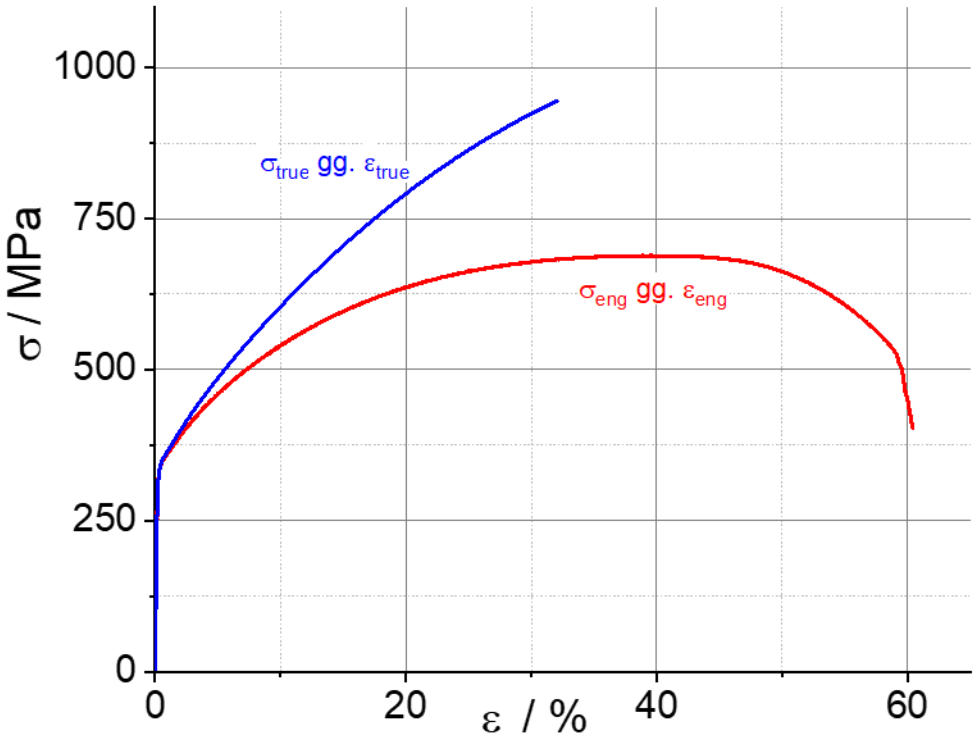
Stability vs. Instability

example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature

- Up to the uniform strain, the tensile test is **stable**.
- Any **perturbation of the cross sectional area** results in **localized plastic deformation due to stress concentration**.
- The localized plastic deformation leads to a **sufficient work-hardening to prevent further localization**.
- Hence, the localization of plastic deformation is stopped by the intrinsic work-hardening.



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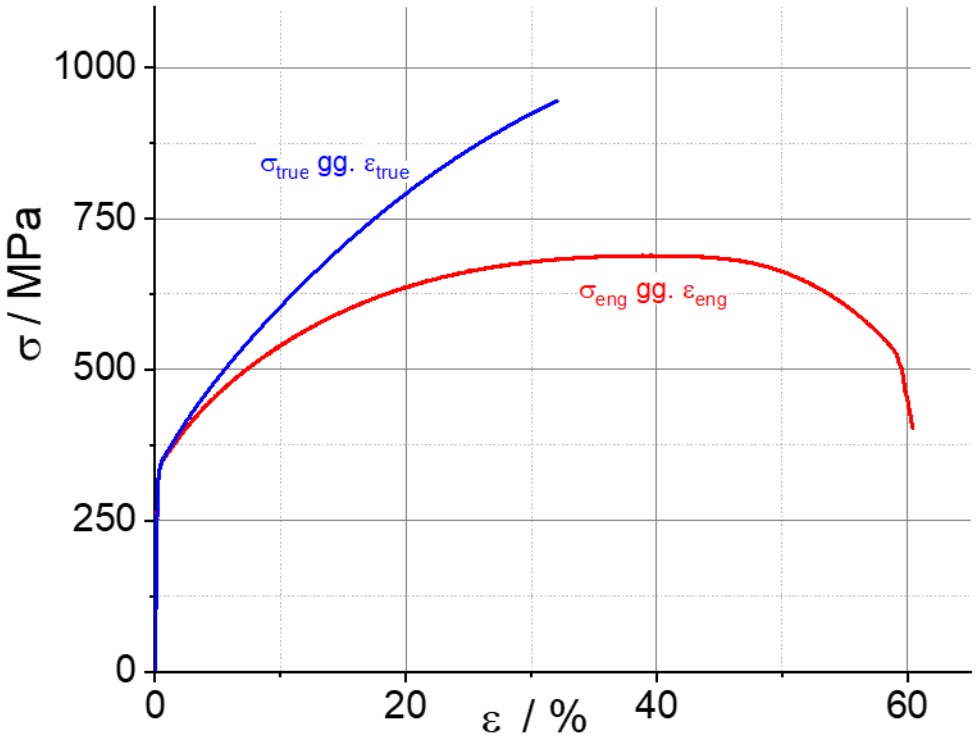
Stability vs. Instability

- Beyond the uniform strain, any perturbation of the cross sectional area still leads to localization of plastic deformation due to stress concentration.
- The **intrinsic work-hardening is not sufficient to prevent the further stress concentration by the further decreasing cross sectional area**. The test is unstable.
- The sample is **only deforming at the weakest perturbation** and **necking occurs**.
  (Note that the perturbation must not only be of geometrical nature, a locally smaller strength can also be point of necking initiation.)

example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature



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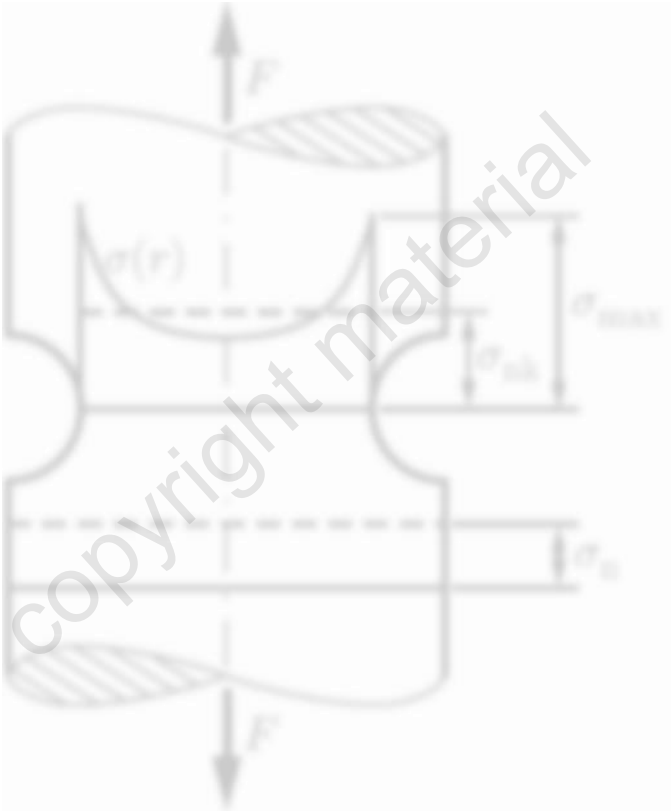
Revision: Notch Effect and Stress Concentration

- Example for force flux in a cylindrical sample with round groove under tension load.
- The flux lines follow the principle stress directions.
- The density of the flux lines corresponds to the local stress.
- **The groove causes a stress concentration. Restricted transverse contraction within the groove region leads to a multiaxial stress state.**

J. Rösler, H. Harders, M. Bäker: „Mechanisches Verhalten der Werkstoffe“, Wiesbaden: Vieweg+Teubner Verlag (2008)

Revision: Notch Effect and Stress Concentration



- Schematic stress (in loading direction) distribution across the cross section.
- σ_{n} is the nominal stress without groove
- σ_{nk} is the nominal stress considering just the reduced cross sectional area in the region of the groove.
- **$\sigma(r)$ is the real stress distribution in axial direction as a function of the radial distance r.**

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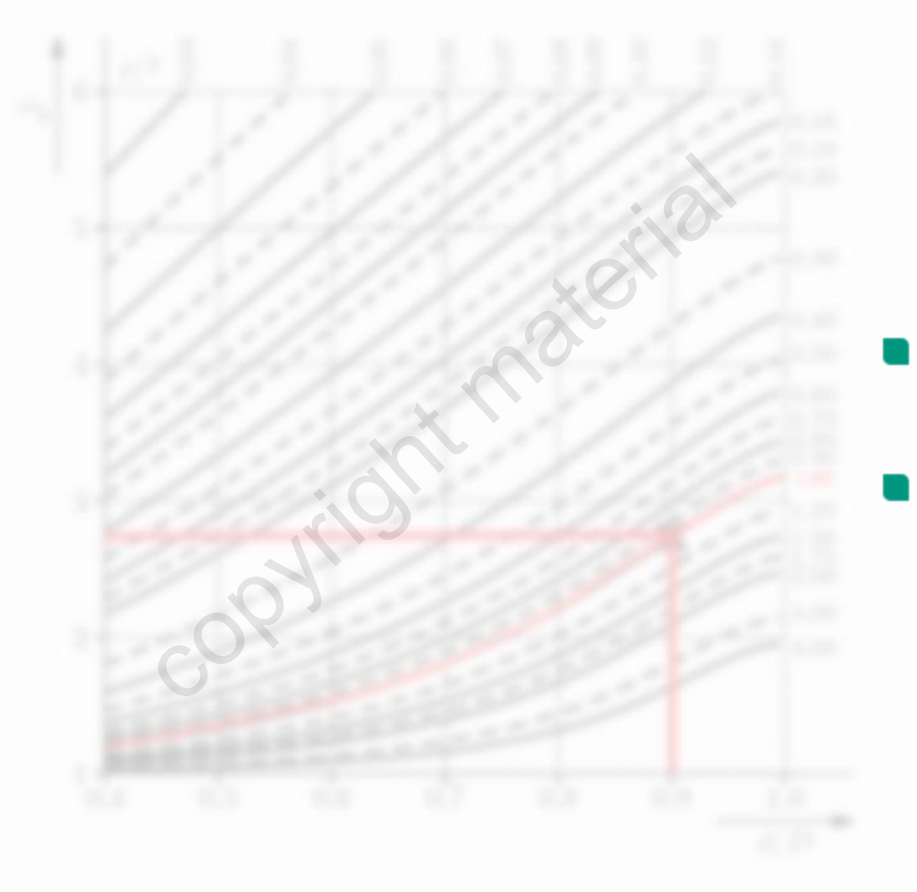
Revision: Notch Effect and Stress Concentration



- The notch number corresponds to the $\alpha_K = \frac{\sigma_{max}}{\sigma_{nk}}$ ratio.
- **The round ($\rho = t$) groove with $90\ \%$ inner diameter results in a stress concentration of about 2.7!**

J. Rösler, H. Harders, M. Bäker: „Mechanisches Verhalten der Werkstoffe“, Wiesbaden: Vieweg+Teubner Verlag (2008)

Stability vs. Instability, Quantitatively

- The stable test requests that the force carried along the sample axis x is constant:

$$F = \sigma_{\text{true}}(x) \cdot A(x) = \text{const.}\ \forall\, x$$

- The force needed for elongation changes during the test with:

$$\frac{\mathrm{d}F}{\mathrm{d}\varepsilon_{\text{true}}} = A \cdot \frac{\mathrm{d}\sigma_{\text{true}}}{\mathrm{d}\varepsilon_{\text{true}}} + \sigma_{\text{true}} \cdot \frac{\mathrm{d}A}{\mathrm{d}\varepsilon_{\text{true}}}$$

$\frac{\mathrm{d}\sigma_{\text{true}}}{\mathrm{d}\varepsilon_{\text{true}}}$… intrinsic work-hardening of the material

$\frac{\mathrm{d}A}{\mathrm{d}\varepsilon_{\text{true}}}$… geometric softening

($\frac{\mathrm{d}A}{\mathrm{d}\varepsilon_{\text{true}}} < 0$ during tensile tests)

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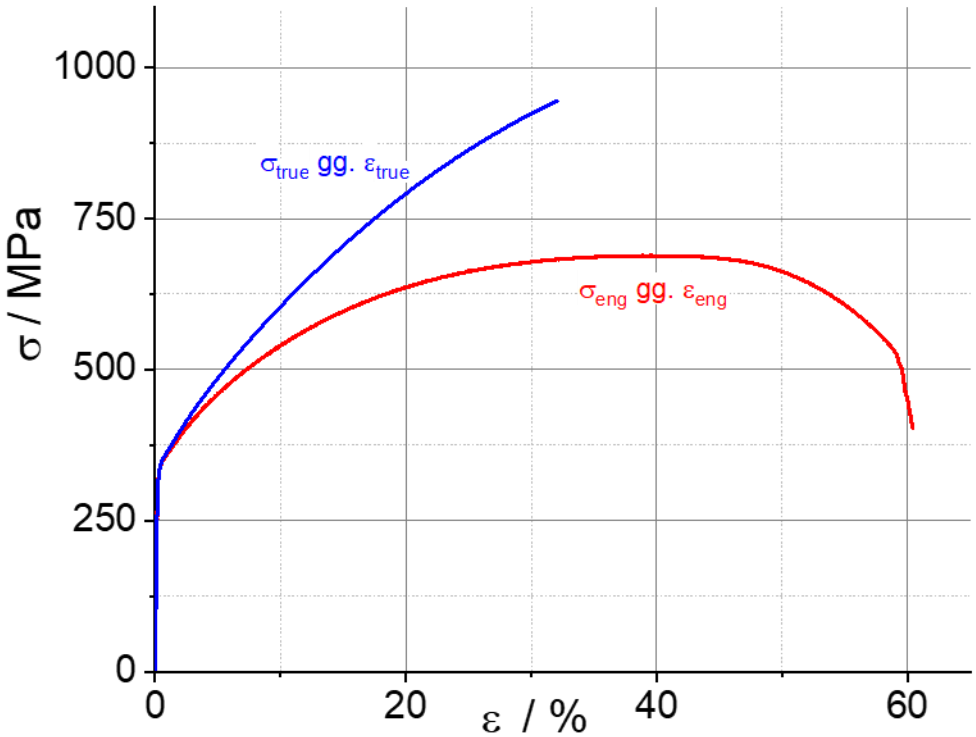

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Stability vs. Instability, Quantitatively

- A stable test request continuously increasing force:

$$\mathrm{d}F > 0$$

$$A \cdot \mathrm{d}\sigma_{\mathrm{true}} + \sigma_{\mathrm{true}} \cdot \mathrm{d}A > 0$$

$$A \cdot \mathrm{d}\sigma_{\mathrm{true}} - \sigma_{\mathrm{true}} \cdot A \cdot \frac{dl}{l} > 0$$

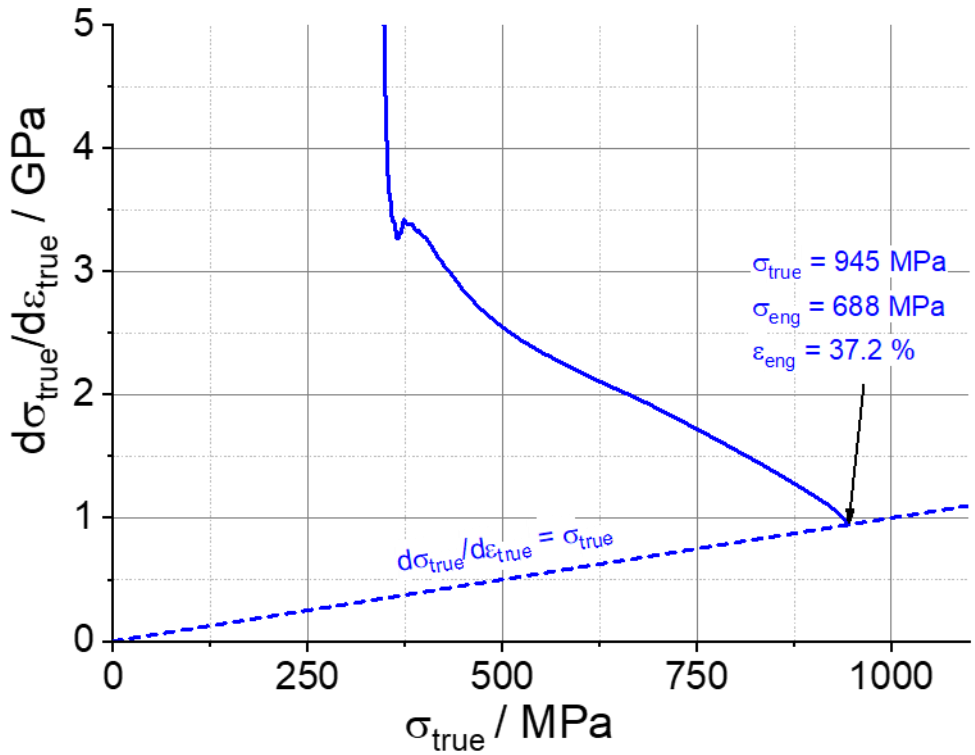
$$\mathrm{d}\sigma_{\mathrm{true}} - \sigma_{\mathrm{true}} \cdot \mathrm{d}\varepsilon_{\mathrm{true}} > 0$$

$$\frac{\mathrm{d}\sigma_{\mathrm{true}}}{\mathrm{d}\varepsilon_{\mathrm{true}}} > \sigma_{\mathrm{true}}$$

Considère criterion

(should always be used for determination of the uniform strain, especially for materials with low work-hardening rates)

example: true work-hardening of CoCrFeMnNi at room temperature for estimation of Considère criterion



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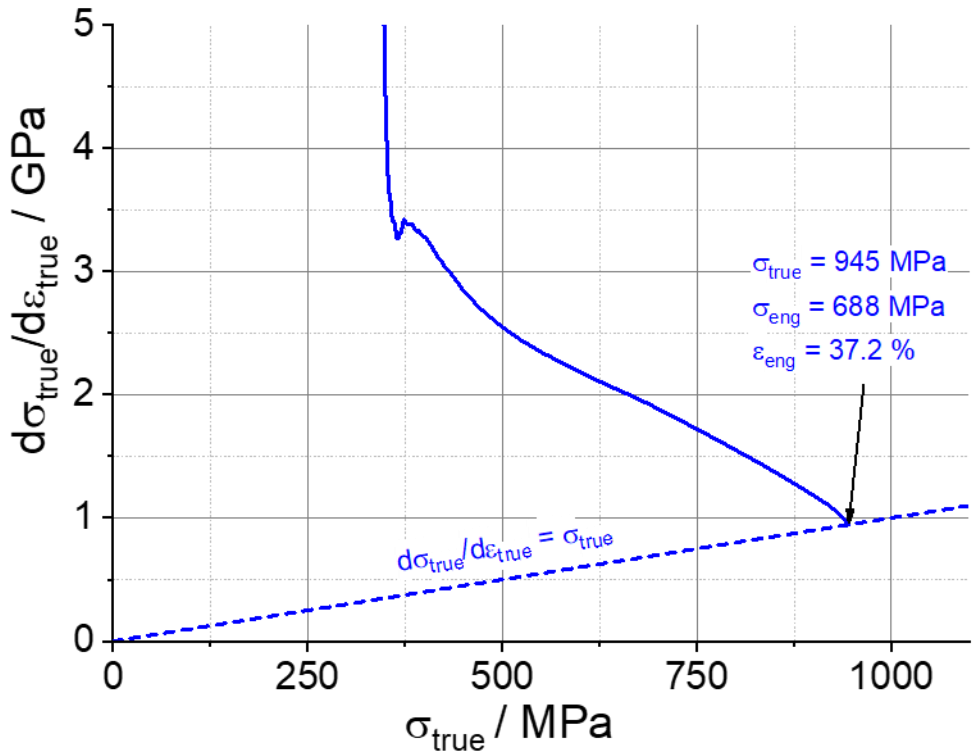
Ductility

- This is counter-intuitive: **high ductility** (high ability to deform plastic) is observed **when work-hardening is high *and* remains high**.
- This is what excels TWIP and TRIP steels due to the dynamic refinement of the microstructure.

example: true work-hardening of CoCrFeMnNi at room temperature for estimation of Considère criterion



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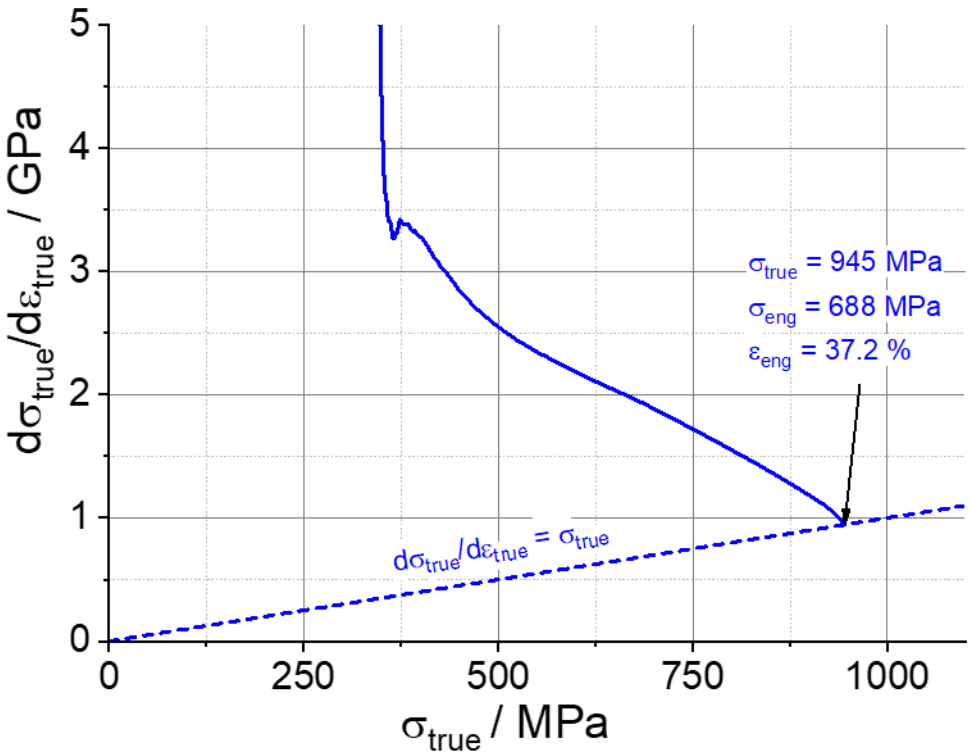
Ductility

- In case mechanical tests are **free of geometrical softening**, e.g. compression tests and torsion tests, **no necking** due to geometrical-mechanical instability occurs!
- The **plastic strains** achieved in these tests are by definition **higher**.

example: true work-hardening of CoCrFeMnNi at room temperature for estimation of Considère criterion



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Ductility

example: true work-hardening of CoCrFeMnNi at room temperature for estimation of Considère criterion

- Since the **size of the necking region only depends on the cross sectional area**, **samples of different length exhibit different strains to fracture while being equally ductile**!
- Hence, **strain to fracture is inappropriate** to judge material's ductility in most cases.
- Comparison of different samples request constant $L/\sqrt{A_0}$ ratio.

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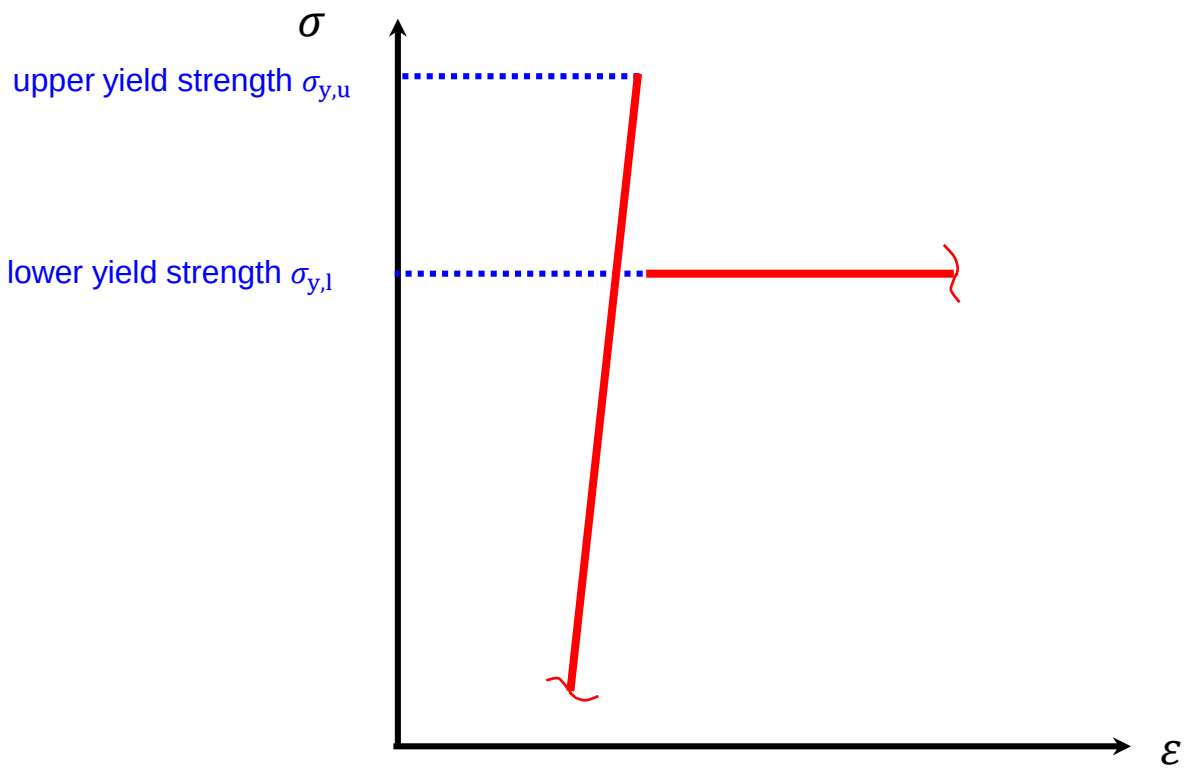
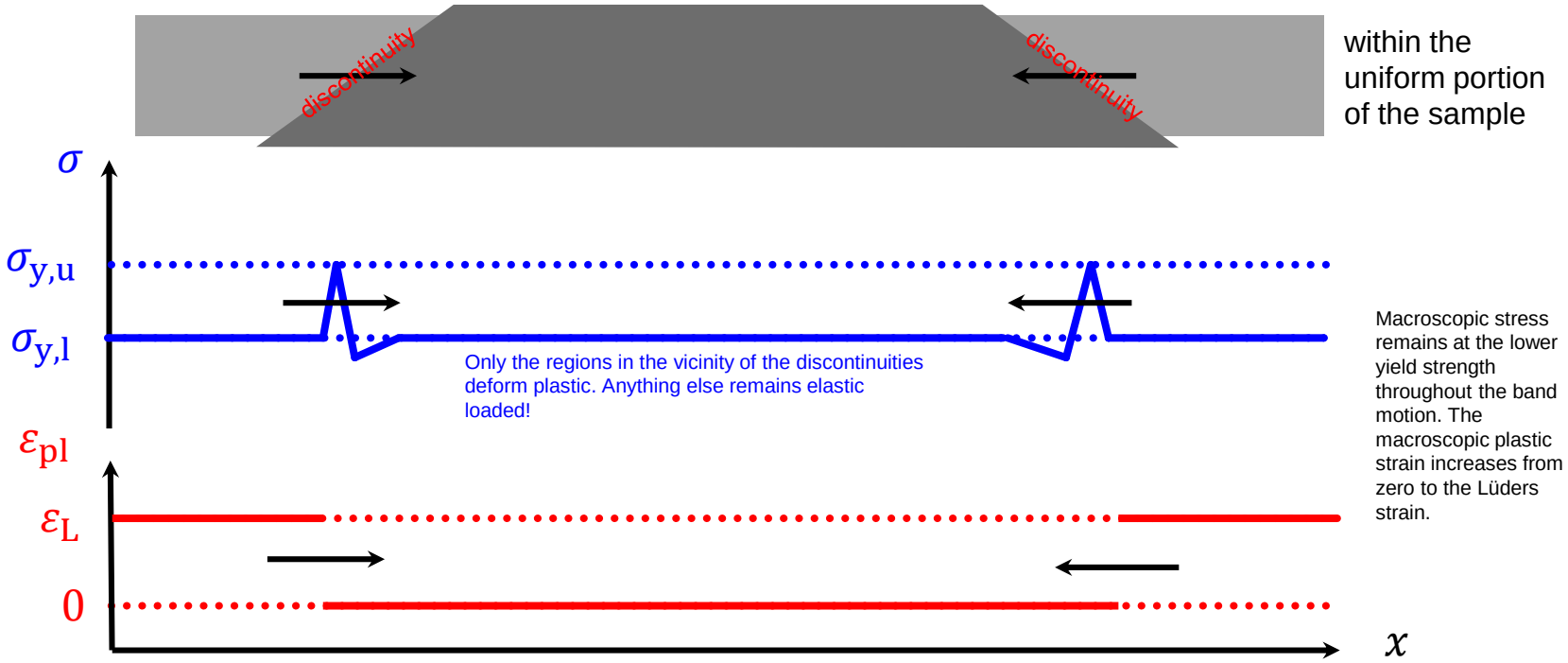
Pronounced Yielding

- Localized deformation does not only contribute to effects at high plastic strains but can also play a vital role in yielding. **Pronounced yielding** occurs when **two distinct strength values** occur: **higher** (onset of any plasticity) and **lower** (maintain plasticity).



Pronounced Yielding

- Discontinuities/perturbations of the cross sectional area (usually at the transition radii) lead to **stress concentrations sufficient to initiate local plastic** deformation by surpassing the upper yield strength.
- The **discontinuities move** through the sample while leaving material behind that is deformed up to the **Lüders strain**.



Pronounced Yielding

- Lüders bands of different nature exist. The commonly known pairs of bands or single bands running trough the entire sample are only one possibility.
- The nature of the band is closely related to the microscopic details of localization and plastic deformation.

example: Lüders band propagation through a shape memory alloy (note the grips for loading and unloading)

https://en.wikipedia.org/wiki/File:L%C3%BCdersband_measured_with_digital_image_correlation_(DIC)_from_LIMESS.gif

Summary

- Simple mechanical tests provide information about **stiffness** (resist. against elastic def.), **strength** (resist. against yielding, certain plastic strain or necking), **work-hardening** (increase of strength during plastic def.), **ductility** (ability to deform plastic), and **toughness** (energy consumption during plastic def.).
- Depending on the loading conditions, **localization of deformation** must be considered in order to describe the macroscopic deformation behavior.