Plasticity

Lecture for “Mechanical Engineering” and “Materials Science and Engineering”
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Topics

- Macroscopic plastic deformation: stress-strain-curve in tensile tests
  - Engineering strain, engineering stress
  - Elastic deformation: stiffness/compliance
  - Elastic-plastic deformation: offset strength
  - Localized plastic deformation I: ultimate tensile strength, uniform strain
  - True strain, true stress, work/strain-hardening
  - Considère criterion
  - Localized deformation II: pronounced yielding and Lüders bands
Revision from Other Lectures

How is a tensile test done?
- Which sample shapes are usually used in tensile tests?
- Which quantities are typically controlled during the tests?
- Which quantities are usually detected?
Metallic materials are very ductile even tough being crystalline!

The chapter will deal with the macroscopic interpretation of simple mechanical tests.

example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature

Stages of Tensile Tests

- Stages of the deformation during the tensile test:
  - **Linear-elastic deformation** (reversible (almost) without internal friction)
  - **Elastic-plastic deformation** (partially reversible, partially irreversible)
  - **Elastic-plastic deformation with necking** (localization of plastic deformation)

- Mostly presented using engineering quantities:
  - **Engin. strain** $\varepsilon_{\text{eng}} = \frac{\Delta l}{l_0}$ (sometimes “total strain”)
  - **Engin. stress** $\sigma_{\text{eng}} = \frac{F}{A_0}$ (sometimes “nominal stress”)

Linear-Elastic Deformation

- The linear-elastic region can be described by Hooke’s law:
  \[ \sigma_{\text{eng}} = E \cdot \varepsilon_{\text{eng}}. \]
- \( E \) is the **resistance against elastic deformation** and called **Young’s modulus**.
- In case of polycrystals, \( E \) corresponds to an averaging of the compliance/stiffness constancies weight by the orientations distribution function (see. Ch. 3).
- \( E \) is one particular stiffness property. Depending on the loading conditions, there are also other stiffness properties, like for example \( G \) with \( \tau = G \cdot \gamma \) or \( K \) with \( p = -K \cdot \frac{\Delta V}{V} \).

Elastic-Plastic Deformation

- In the elastic-plastic region, some portion of the deformation is irreversible.

- The onset of plastic deformation is called **yield strength** $\sigma_y$ and determines the **resistance against plastic deformation**.

- In the case of rather continuous stress-strain curves **without pronounced yielding**, **offset strength** is used to describe onset of plastic deformation. For example $\sigma_{0.2}$ is the **resistance against 0.2% plastic strain**.

- Anyhow, both quantities indicate macroscopic strength. Microscopic plasticity is taking place also at stresses below $\sigma_y$ and of course $\sigma_{0.2}$.

Elastic-Plastic Deformation

- During unloading, the elastic deformation *almost* resets.
- Onset of plasticity during re-loading is observed at a higher stress in comparison to the initially undeformed material. This is called strain- or work-hardening.

* Bauschinger effect

Necking

- **Ultimate tensile strength** $\sigma_m$ determines the **maximum engineering stress** that the sample can resist. Beyond $\sigma_m$, the sample exhibits **necking** and the plastic deformation becomes **localized** to a small portion of the gauge length.

- Plastic strain which achieved up to $\sigma_m$ is called **uniform elongation** (common but imprecise; better to use uniform strain).

- Necking occurs due to the **mechanical-geometrical instability** of the tensile test.

*example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature*

True Stress, True Strain

- For further analysis, the introduction of instantaneous stresses and strains is useful, true strain and true stress:
  - \( d\varepsilon_{\text{true}} = \frac{dl}{l} \), hence, \( \varepsilon_{\text{true}} = \ln \frac{l}{l_0} \)
  - \( \sigma_{\text{true}} = \frac{F}{A} \)

- Assuming uniform elongation (prismatic sample shape) and volume conservation:
  - \( V = A_0 \cdot l_0 = A \cdot l = \text{const.} \)
  - hence, \( A = \frac{A_0}{1 + \varepsilon_{\text{eng}}} \)

  gives
  - \( \sigma_{\text{true}} = \sigma_{\text{eng}} (1 + \varepsilon_{\text{eng}}) \)
  - \( \varepsilon_{\text{true}} = \ln (1 + \varepsilon_{\text{eng}}) \)

Note that true strains can be added while engineering strains cannot.

**True Stress, True Strain**

Direct comparison of $\varepsilon_{\text{true}}$ and $\varepsilon_{\text{eng}}$ as a function of $\varepsilon_{\text{eng}}$.

- Compression testing
- Tensile testing

\[ \begin{align*}
\varepsilon_{\text{true}} & \quad \varepsilon_{\text{eng}} \\
-1.61 & \quad -0.92 \quad -0.51 \quad -0.22 \quad 0.00 \\
0.69 & \quad 1.10
\end{align*} \]
True Stress, True Strain

Direct comparison of $\sigma_{\text{true}}$ and $\sigma_{\text{eng}}$ as a function of $\varepsilon_{\text{eng}}$.

Compression testing

Tensile testing

Plasticity
Volume conservation is an assumption not a prerequisite, also for plastic deformation!

Strictly, volume conservation requests a Poisson's ratio of $\nu = 0.5$. For the elastic region, $\nu$ is typically about 0.3, after yielding it continuously increases to 0.5. The deviations are usually neglected when converting from engineering to true quantities.

In the necking region, the analytical conversion of engineering to true quantities is not possible anymore. You need to locally resolve strain and stress, for example by DIC.

Stability vs. Instability

- Loading of a sample in tensile geometry leads to two different contributions to the force needed:
  - **Geometrical softening**: during loading, the cross section of the sample becomes smaller. In case of nil resistance against further deformation, the force needed to deform would become smaller.
  - **Intrinsic strengthening**: The material itself work-hardens; its yield stress is continuously increasing.

*example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature*

Stability vs. Instability

- Up to the uniform strain, the tensile test is **stable**.
- Any **perturbation of the cross sectional area** results in localized plastic deformation due to stress concentration.
- The localized plastic deformation leads to a **sufficient work-hardening** to prevent further localization.
- Hence, the localization of plastic deformation is stopped by the intrinsic work-hardening.

example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature

Stability vs. Instability

- Beyond the uniform strain, any perturbation of the cross sectional area still leads to localization of plastic deformation due to stress concentration.

- The intrinsic work-hardening is not sufficient to prevent the further stress concentration by the further decreasing cross sectional area. The test is unstable.

- The sample is only deforming at the weakest perturbation and necking occurs. (Note that the perturbation must not only be of geometrical nature, a locally smaller strength can also be point of necking initiation.)

Revision: Notch Effect and Stress Concentration

- Example for force flux in a cylindrical sample with round groove under tension load.
- The flux lines follow the principle stress directions.
- The density of the flux lines corresponds to the local stress.
- **The groove causes a stress concentration.** Restricted transverse contraction within the groove region leads to a multiaxial stress state.

Revision: Notch Effect and Stress Concentration

- Schematic stress (in loading direction) distribution across the cross section.
- \( \sigma_n \) is the nominal stress without groove
- \( \sigma_{nk} \) is the nominal stress considering just the reduced cross sectional area in the region of the groove.
- \( \sigma(r) \) is the real stress distribution in axial direction as a function of the radial distance \( r \).

The notch number corresponds to the $\alpha_K = \frac{\sigma_{\text{max}}}{\sigma_{nk}}$ ratio.

The round ($\rho = t$) groove with 90% inner diameter results in a stress concentration of about 2.7!
Stability vs. Instability, Quantitatively

- The stable test requests that the force carried along the sample axis $x$ is constant:

$$F = \sigma_{\text{true}}(x) \cdot A(x) = \text{const. \ \forall \ x}$$

- The force needed for elongation changes during the test with:

$$\frac{dF}{d\varepsilon_{\text{true}}} = A \cdot \frac{d\sigma_{\text{true}}}{d\varepsilon_{\text{true}}} + \sigma_{\text{true}} \cdot \frac{dA}{d\varepsilon_{\text{true}}}$$

- Intrinsic work-hardening of the material
- Geometric softening

Example: tensile test conducted on face centered cubic, polycrystalline CoCrFeMnNi at room temperature

Stability vs. Instability, Quantitatively

- A stable test request continuously increasing force:

\[ dF > 0 \]
\[ A \cdot d\sigma_{\text{true}} + \sigma_{\text{true}} \cdot dA > 0 \]
\[ A \cdot d\sigma_{\text{true}} - \sigma_{\text{true}} \cdot A \cdot \frac{dl}{l} > 0 \]
\[ d\sigma_{\text{true}} - \sigma_{\text{true}} \cdot d\varepsilon_{\text{true}} > 0 \]
\[ \frac{d\sigma_{\text{true}}}{d\varepsilon_{\text{true}}} > \sigma_{\text{true}} \]

**Considère criterion**

(should always be used for determination of the uniform strain, especially for materials with low work-hardening rates)

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**example:** true work-hardening of CoCrFeMnNi at room temperature for estimation of Considère criterion

- \( \sigma_{\text{true}} = 945 \text{ MPa} \)
- \( \sigma_{\text{eng}} = 688 \text{ MPa} \)
- \( \varepsilon_{\text{eng}} = 37.2 \% \)

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Ductility

- This is counter-intuitive: high ductility (high ability to deform plastic) is observed when work-hardening is high and remains high.
- This is what excels TWIP and TRIP steels due to the dynamic refinement of the microstructure.

Ductility

- In case mechanical tests are free of geometrical softening, e.g. compression tests and torsion tests, no necking due to geometrical-mechanical instability occurs!
- The plastic strains achieved in these tests are by definition higher.

example: true work-hardening of CoCrFeMnNi at room temperature for estimation of Considère criterion
Ductility

- Since the size of the necking region only depends on the cross sectional area, samples of different length exhibit different strains to fracture while being equally ductile!
- Hence, strain to fracture is inappropriate to judge material’s ductility in most cases.
- Comparison of different samples request constant $L/\sqrt{A_0}$ ratio.

example: true work-hardening of CoCrFeMnNi at room temperature for estimation of Considère criterion

Localized deformation does not only contribute to effects at high plastic strains but can also play a vital role in yielding. **Pronounced yielding** occurs when two distinct strength values occur: higher (onset of any plasticity) and lower (maintain plasticity).
Pronounced Yielding

- Discontinuities/perturbations of the cross sectional area (usually at the transition radii) lead to stress concentrations sufficient to initiate local plastic deformation by surpassing the upper yield strength.
- The **discontinuities move** through the sample while leaving material behind that is deformed up to the **Lüders strain**.
Pronounced Yielding

- Lüders bands of different nature exist. The commonly known pairs of bands or single bands running through the entire sample are only one possibility.
- The nature of the band is closely related to the microscopic details of localization and plastic deformation.

example: Lüders band propagation through a shape memory alloy (note the grips for loading and unloading)

https://en.wikipedia.org/wiki/File:L%C3%BCdersband_measured_with_digital_image_correlation_(DIC)_from_LIMESS.gif
Summary

Simple mechanical tests provide information about **stiffness** (resist. against elastic def.), **strength** (resist. against yielding, certain plastic strain or necking), **work-hardening** (increase of strength during plastic def.), **ductility** (ability to deform plastic), and **toughness** (energy consumption during plastic def.).

Depending on the loading conditions, **localization of deformation** must be considered in order to describe the macroscopic deformation behavior.