Karlsruhe Institute of Technology

Institute for Applied Materials

Plasticity

Lecture for “Mechanical Engineering” and “Materials Science and Engineering”
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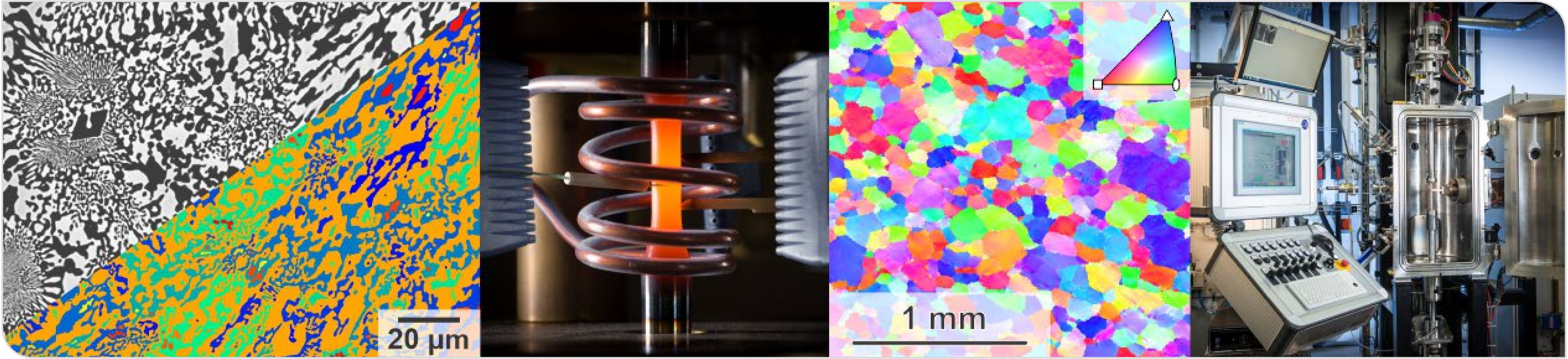


Topics

- Equivalent Stresses
  - Maximum Normal Stress Theory (Rankine)
  - Maximum Shear Stress Theory (Tresca)
  - Maximum Distortion Strain Energy Theory (v. Mises)
- Work-hardening
  - Isotropic Hardening
  - Kinematic Hardening
- Plastic Flow

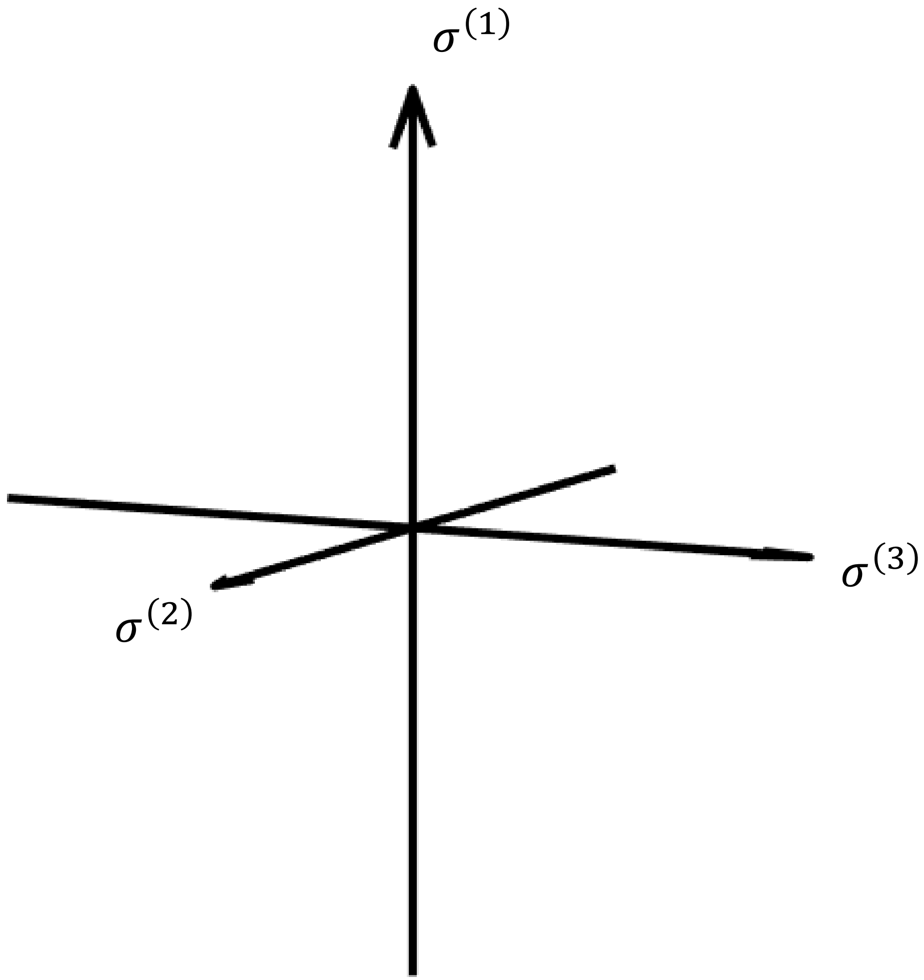
Equivalent Stresses

- As we have seen, **stress states are described by tensors** and **mechanical testing only provides scalar mechanical properties**.
- Hence, there is a **need for hypotheses on how multiaxial stress states initiate failure**. What is actually considered failure depends on the needs: it can be fracture, onset of plastic deformation, small plastic strain, necking, etc.
- Since the **failure criterion cannot depend on the choice of the coordinate system** by the observer, all following **theories utilize scalar invariants of tensors**, equivalent stresses σ_V, in comparison to scalars σ_c determined in mechanical tests!
- **Design criterion for parts is then $\sigma_\mathrm{V} < \sigma_\mathrm{c}$** and the **onset of failure** occurs **at $\sigma_\mathrm{V} = \sigma_\mathrm{c}$**.
- In general, any combination of invariants could be utilized for equivalent stress hypotheses but there are some which prevailed due to their physical basis.

Maximum Normal Stress Theory (Rankine)

- **Normal stress theory** is utilized for brittle materials. It is assumed that materials do not deform plastic and **failure occurs by cleavage (normal stress induced fracture)**.
- In case that $\sigma^{(1)}$, $\sigma^{(2)}$ and $\sigma^{(3)}$ are the principle stresses of a stress state σ_{ik}, maximum normal stress is given by the largest principle stress: $\sigma_{\mathrm{V}} = \max(|\sigma^{(1)}|, |\sigma^{(2)}|, |\sigma^{(3)}|)$.
- **The critical scalar from mechanical testing is the cleavage stress.**
- In engineering, the principle stresses are typically assigned in an ordered way: $\sigma^{(1)} > \sigma^{(2)} > \sigma^{(3)}$.

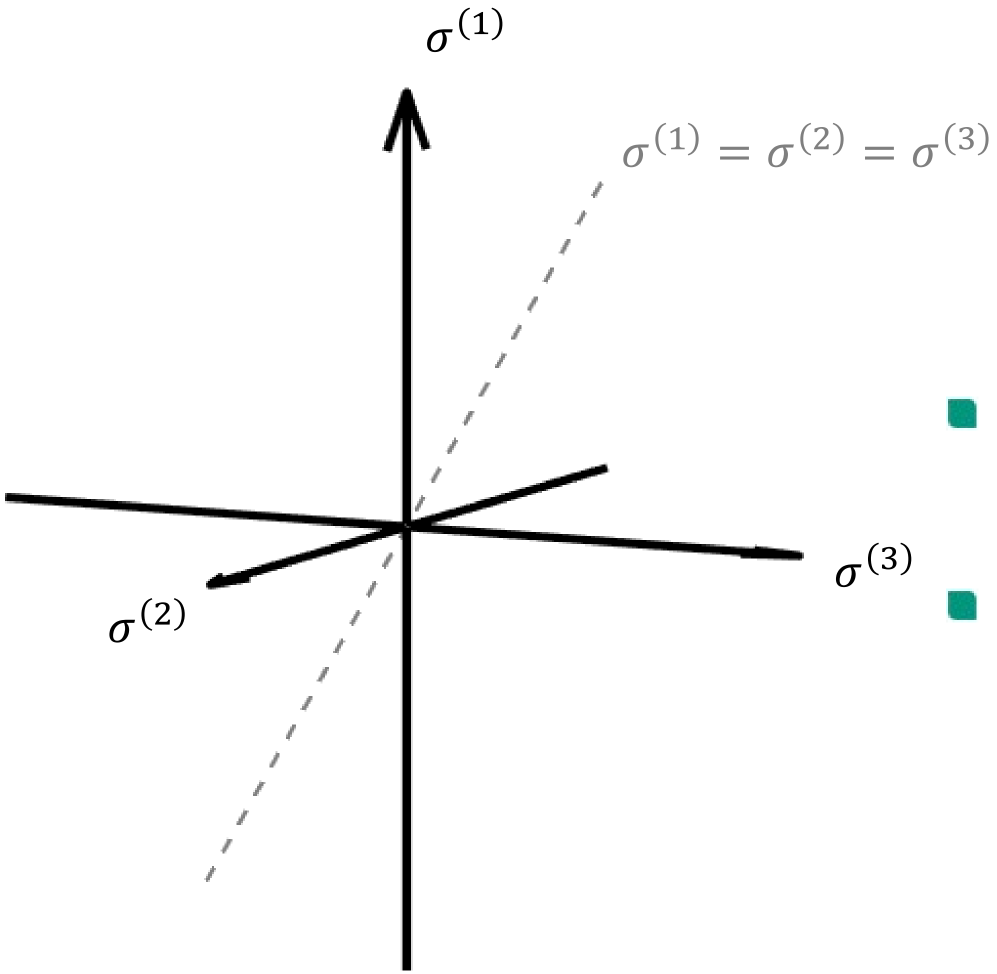
Maximum Normal Stress Theory (Rankine)



- We will use the principle stress space for visualization with

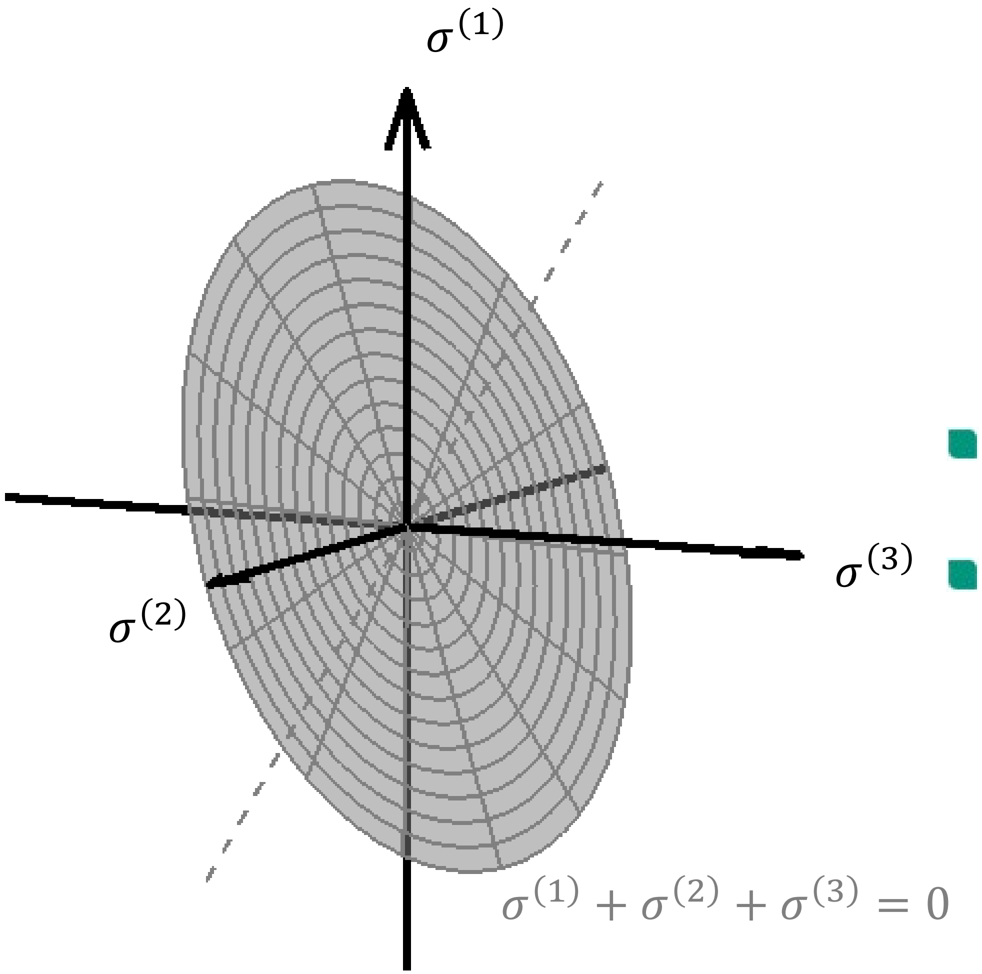
  $$\sigma^{(1)} > \sigma^{(2)} > \sigma^{(3)}$$

Maximum Normal Stress Theory (Rankine)



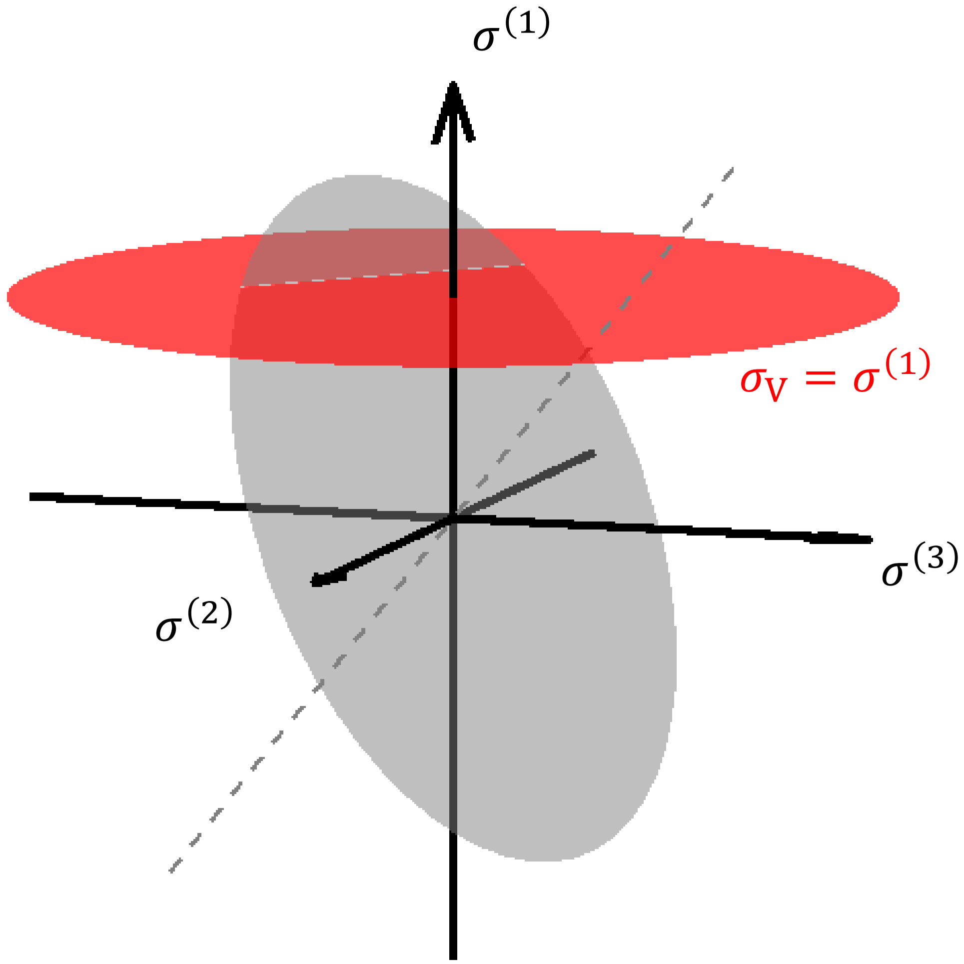
- In case of an hydrostatic stress state: $\sigma^{(1)} = \sigma^{(2)} = \sigma^{(3)}$.
- This is a straight line in principle stress space.

Maximum Normal Stress Theory (Rankine)



- In case of a deviatoric stress state: $\sigma^{(1)} + \sigma^{(2)} + \sigma^{(3)} = 0$.
- This is a plane perpendicular to the hydrostatic straight line.

Maximum Normal Stress Theory (Rankine)

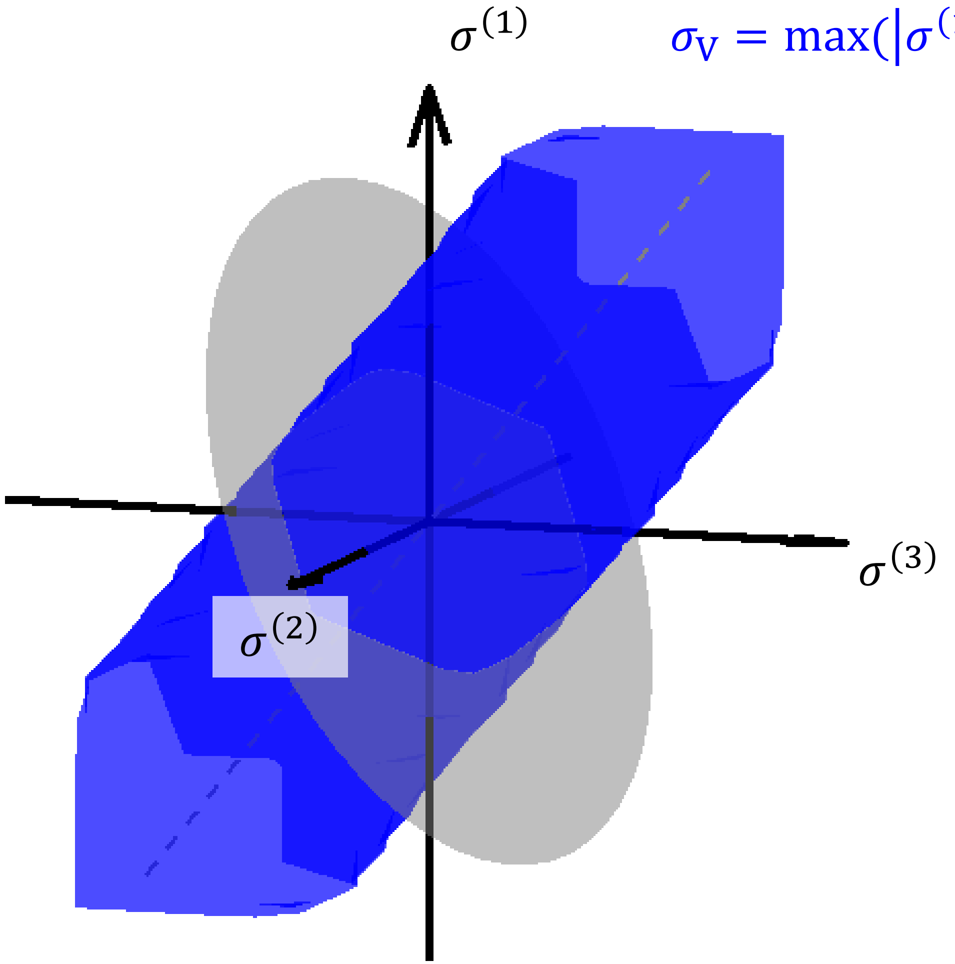


- The maximum normal stress is a plane intersecting at $\sigma_V = \sigma^{(1)} = \sigma_c$.
- Strictly, cleavage under compression load is not possible.
- However, higher critical stresses for special types of fracture (shear compression fracture) are also sometimes assumed for compression load.

Maximum Shear Stress Theory (Tresca)

- It became clear very early that metallic materials exhibit plastic deformation when exposed to shear loading.
- **Maximum shear stress** of a stress state corresponds to the **maximum difference of the principle stress**: $\sigma_{\mathrm{V}} = \max\left(\left|\sigma^{(1)} - \sigma^{(2)}\right|, \left|\sigma^{(1)} - \sigma^{(3)}\right|, \left|\sigma^{(2)} - \sigma^{(3)}\right|\right)$.
- **The critical scalar from mechanical testing is the yield strength.**

Maximum Shear Stress Theory (Tresca)



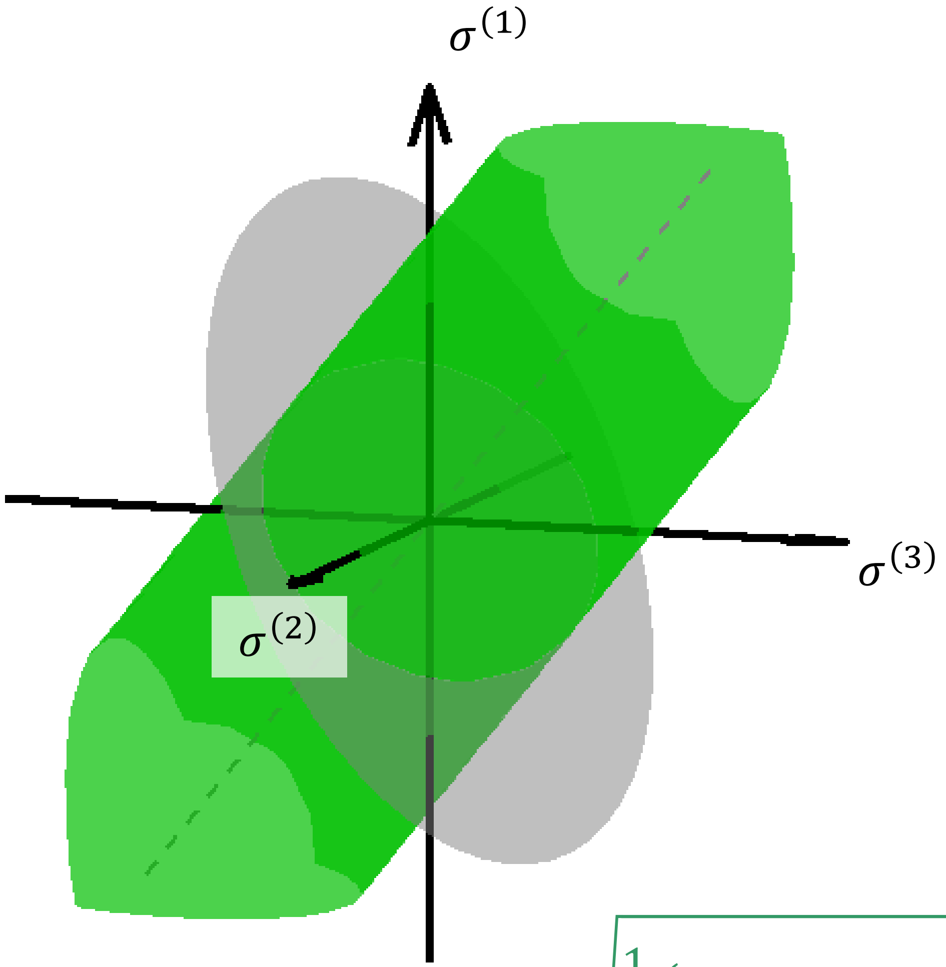
$$\sigma_{\mathrm{V}} = \max(|\sigma^{(1)} - \sigma^{(2)}|, |\sigma^{(1)} - \sigma^{(3)}|, |\sigma^{(2)} - \sigma^{(3)}|)$$

- The surface defined by the respective yield criterion $\sigma_{\mathrm{V}} = \sigma_{\mathrm{c}}$ is called yield surface.
- The yield surface of the maximum shear theory is a **cylinder of hexagonal cross section parallel to the hydrostatic straight line**.
- Hydrostatic contributions do not lead to yielding.

Max. Distortion Strain Energy Theory (v. Mises)

- It is assumed that the material exhibits **plastic deformation once a critical distortion strain energy is achieved**.
- In most cases, very good agreement is found for the onset of plastic deformation in metallic materials. Therefore, v. Mises criterion is the standard theory for design in mechanical engineering.
- The equivalent stress is as follows: $\sigma_V = \sqrt{\frac{1}{2}\left((\sigma^{(1)} - \sigma^{(2)})^2 + (\sigma^{(1)} - \sigma^{(3)})^2 + (\sigma^{(2)} - \sigma^{(3)})^2\right)} = \sqrt{\frac{1}{2}\left((\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2\right) + 3(\tau_{12}^2 + \tau_{23}^2 + \tau_{13}^2)}$
- **The critical scalar from mechanical testing is the yield strength.**

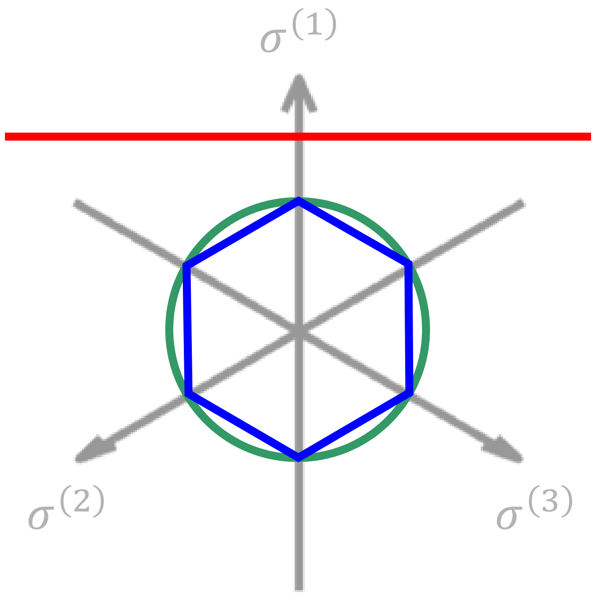
Max. Distortion Strain Energy Theory (v. Mises)



- The yield surface of the v. Mises theory is a **cylinder of circular cross section parallel to the hydrostatic straight line**.
- Hydrostatic contributions do not lead to yielding.

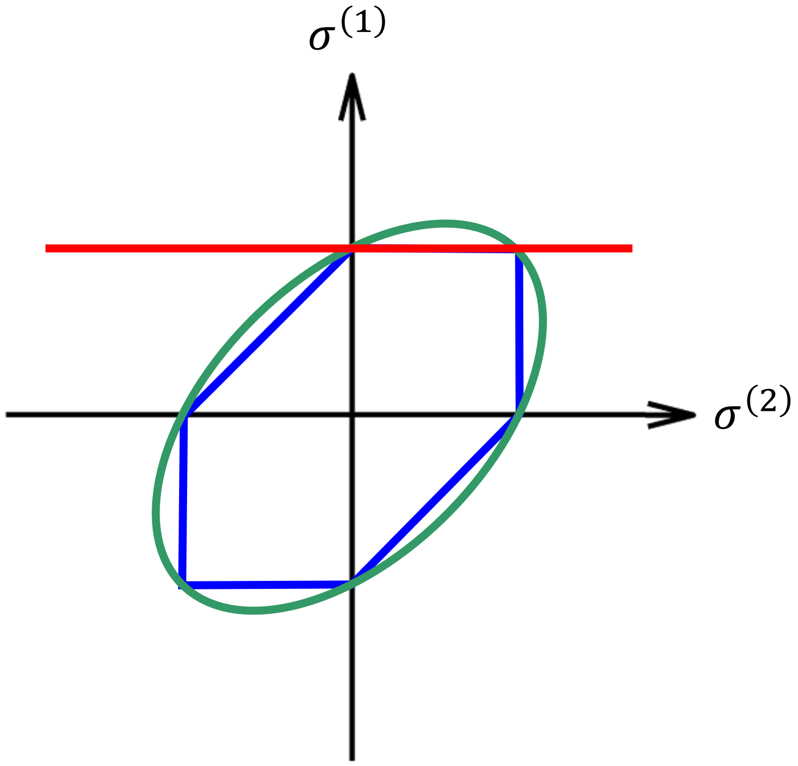
$$\sigma_V = \sqrt{\frac{1}{2}\left((\sigma^{(1)}-\sigma^{(2)})^2+(\sigma^{(1)}-\sigma^{(3)})^2+(\sigma^{(2)}-\sigma^{(3)})^2\right)}$$

Comparison



- This is the projection into the deviatoric plane (view along the hydrostatic straight line): $\sigma^{(1)} + \sigma^{(2)} + \sigma^{(3)} = 0$.

Comparison

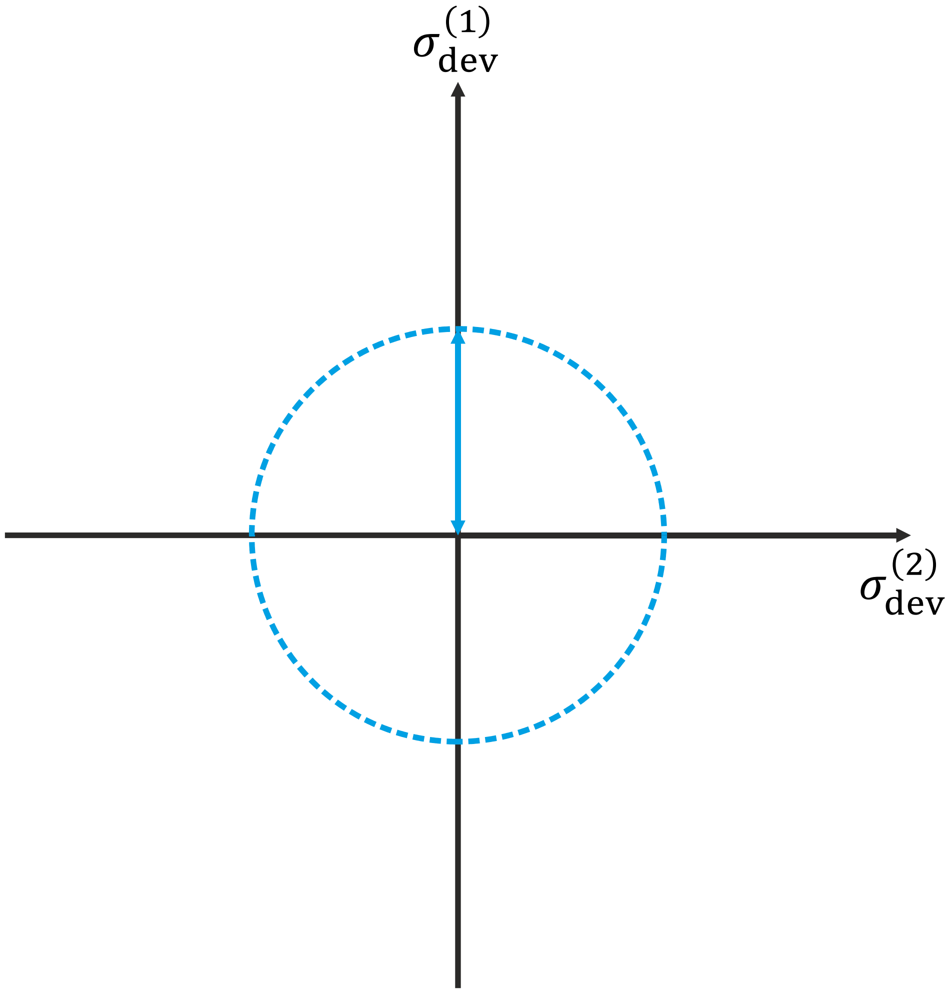


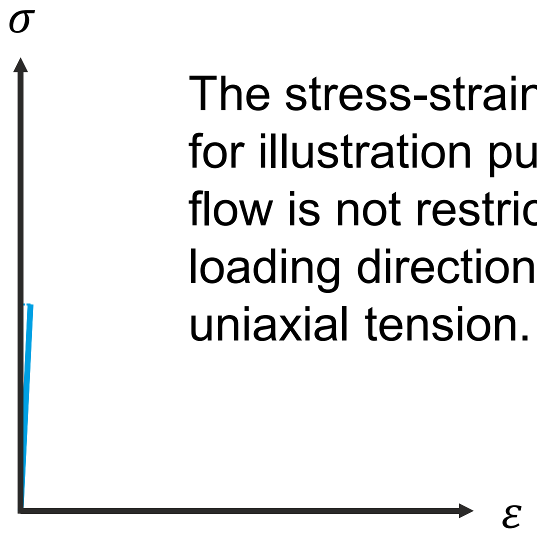
- For a plane stress state $\sigma^{(3)} = 0$, the projection becomes distorted.

Hardening

- The aforementioned theories describe the onset of plasticity under complex, multiaxial loading conditions. All introduced theories describe isotropic onset of plasticity.
- As shown in Ch. 2, metallic materials exhibit a **more or less pronounced work-hardening during plastic deformation**. Hence, the critical value for onset of plasticity increases with increasing plastic strain. This has to be considered when materials are upset or in case of failure analysis.
- There are two fundamental possibilities of hardening which can be superimposed:
  - **Isotropic**: The hardening **does not depend on the previous plastic deformation**, the solid was exposed to. The **yield surface increases isotropic** in principle stress space.
  - **Kinematic**: The hardening **depends on the direction of the plastic deformation**, the solid was exposed to. The **yield surface shifts** in principle stress space.

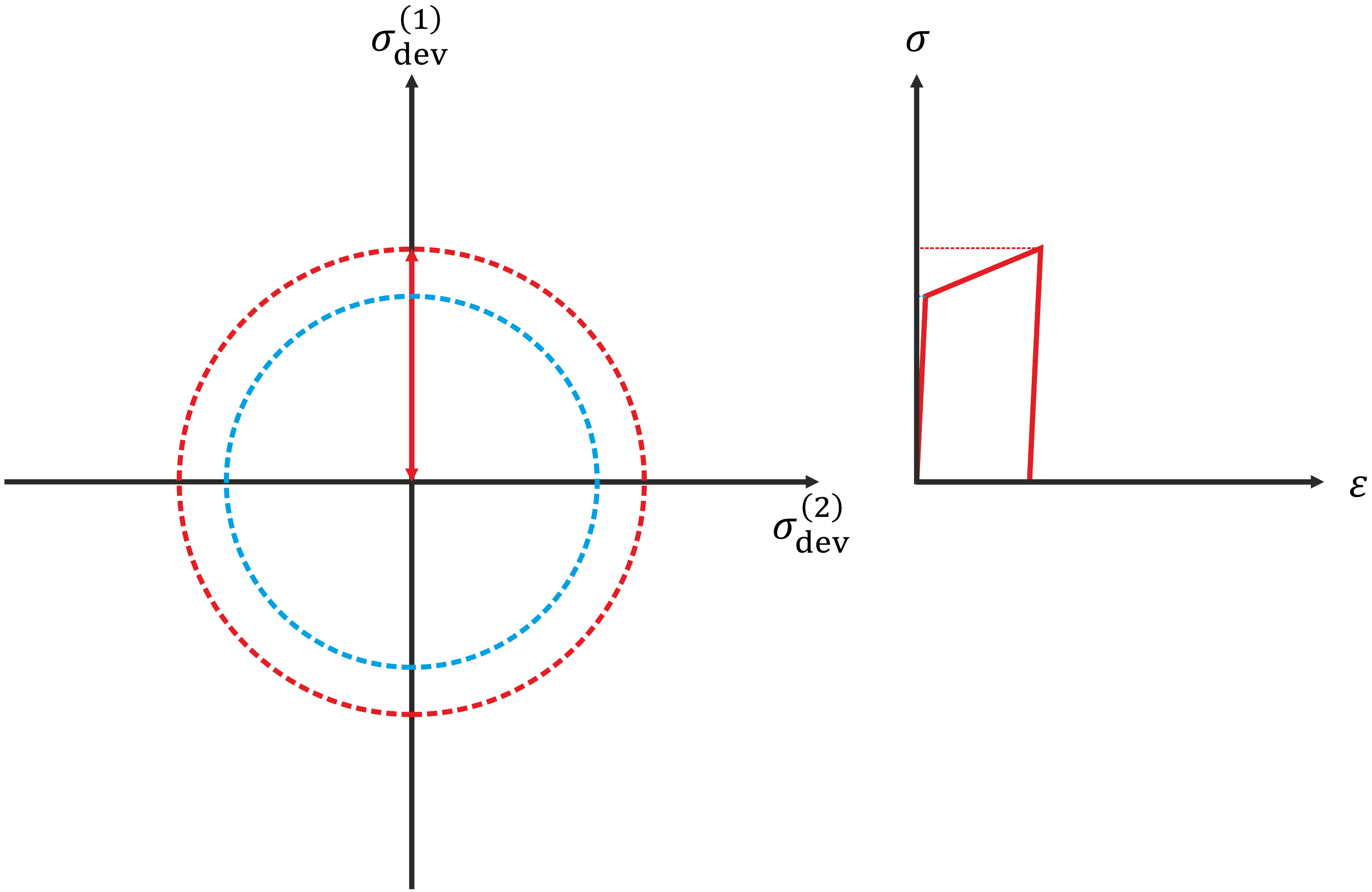
Isotropic Hardening



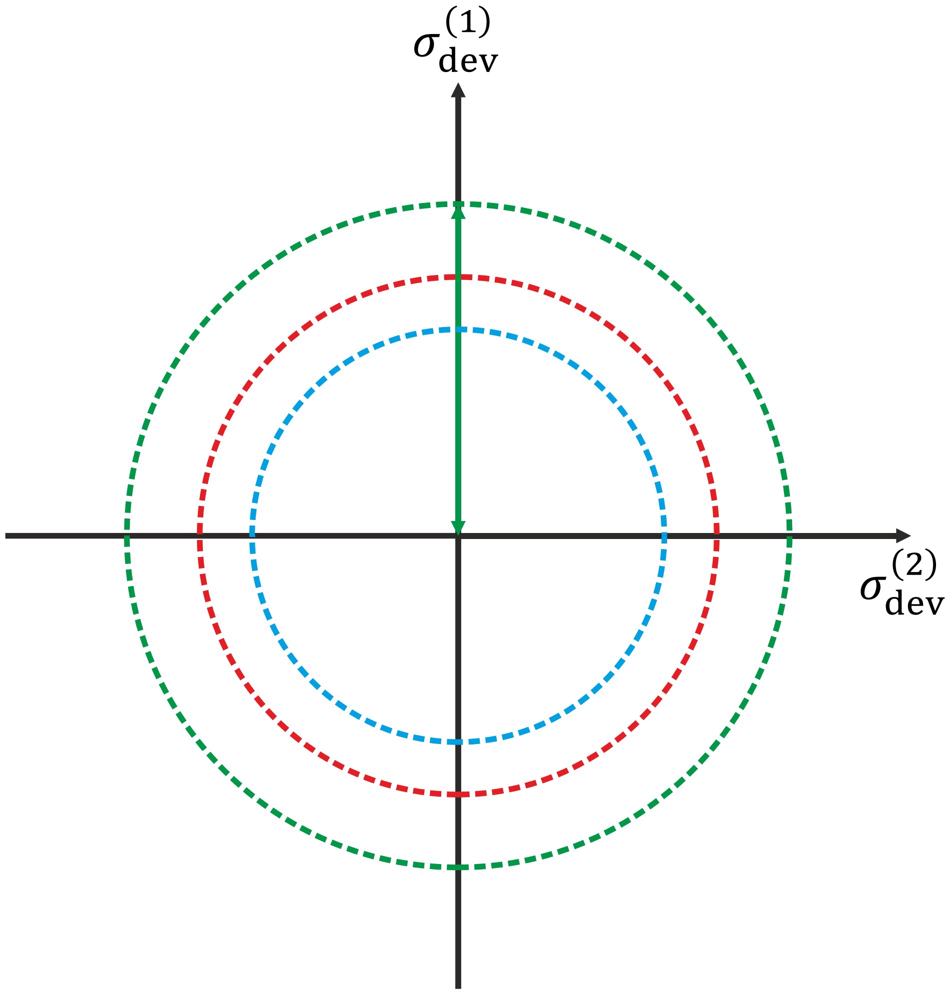


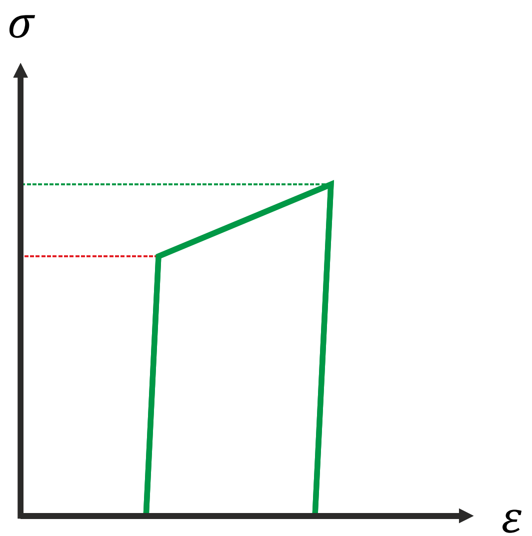
The stress-strain curve is just for illustration purpose. Plastic flow is not restricted to the loading direction under uniaxial tension.

Isotropic Hardening

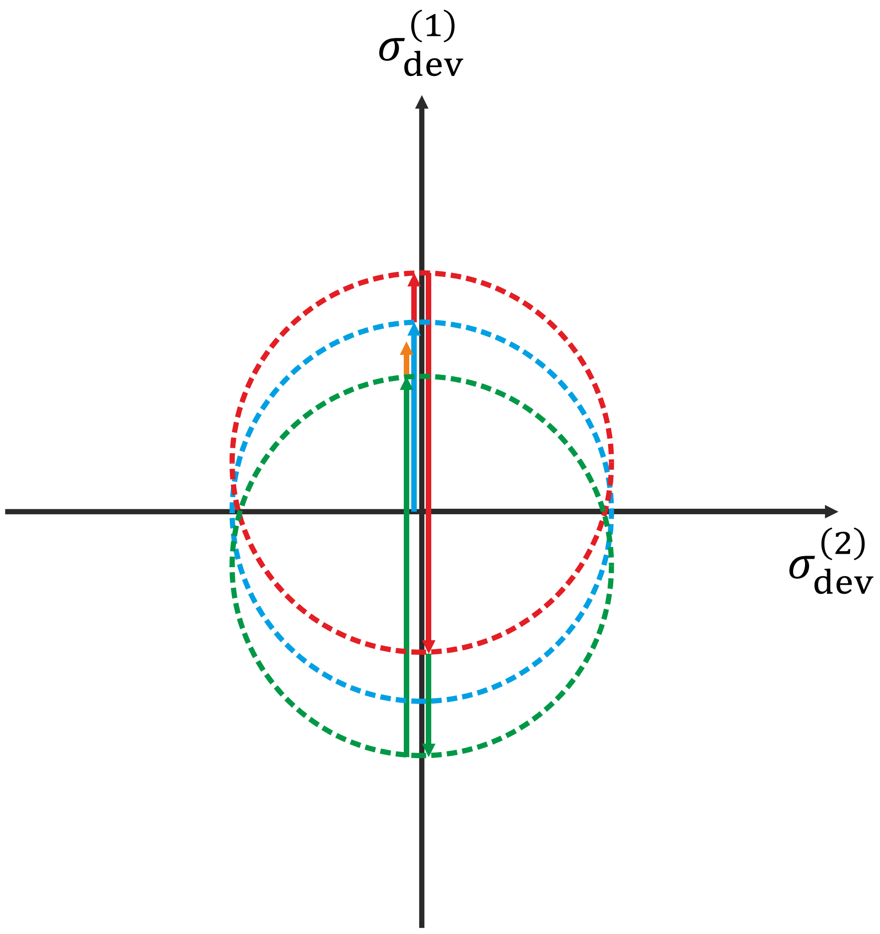


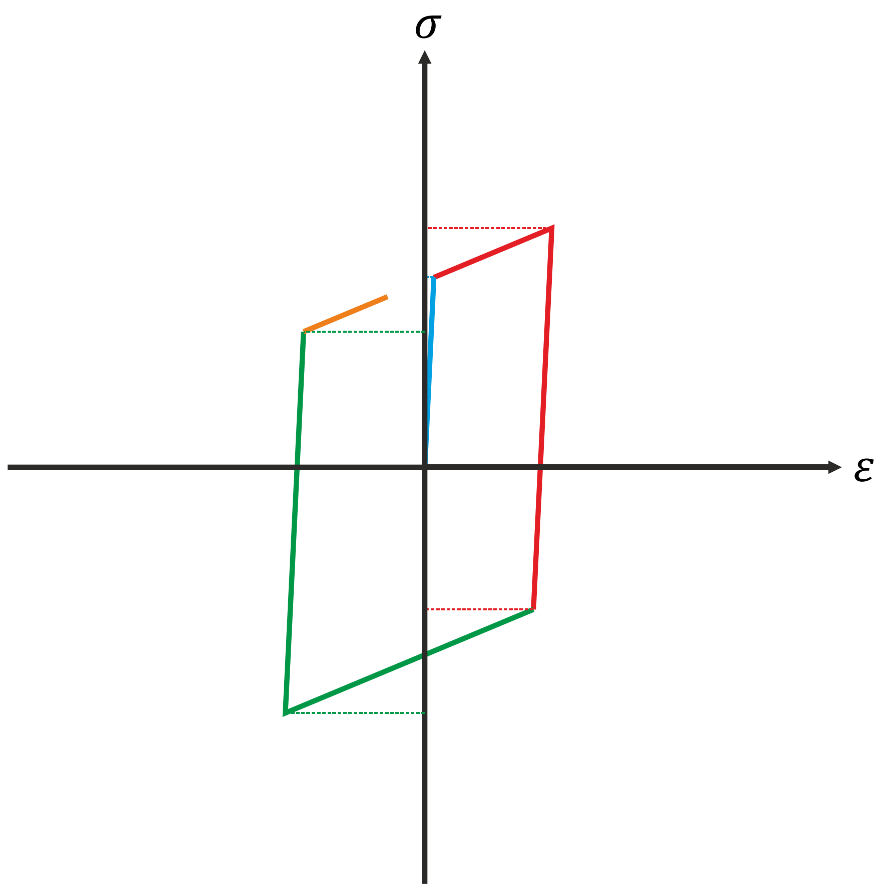
Isotropic Hardening





Kinematic Hardening





Plastic Flow

- In the previous slides, criteria for the onset of plasticity under multiaxial stress states and principles for the hardening behavior were introduced: The questions of when the yield surface is reached and how it changes during plastic flow are described.
- In order to fully describe the macroscopic plastic deformation of materials, **assumptions about the extent and direction of plastic strain release** are necessary, so-called **flow rules**. This addresses the question how the solid deforms when the yield surface is reached.
- When plastic deformation is initiated, there is no biunique correlation of stress and strain anymore. Rather, the **current state of the solid depends on the history of straining**.
- Hence, most flow theories, utilize **incremental description of plastic** deformation: $\mathrm{d}\varepsilon_{ik}^{\mathrm{pl}} = \mathrm{d}\varepsilon_{ik}^{\mathrm{pl}}(\sigma_{ik})$.
- The plastic strain rate ${}^{\mathrm{d}\varepsilon_{ik}^{\mathrm{pl}}}/_{\mathrm{dt}} = \dot{\varepsilon}_{ik}^{\mathrm{pl}}(\sigma_{ik})$ is usually used in order to avoid an entire differential description. In case of the by far mostly employed theories, **associated flow**, the **direction of the plastic strain rate is perpendicular to the yield surface**. The magnitude (plastic multiplier) depends on the respective theory.

Summary

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- **Equivalent stress theories** allow for the **determination of the onset of plastic deformation under multiaxial stress states**.
- **Hardening rules** allow for the **description of changing yield surfaces** due to multiple work-hardening processes **over the course of plastic deformation**.
- **Flow rules** describe the **plastic flow initiated when yield surface is reached**.